

Essays on Asymmetric Labor-Market Fluctuations and Economic Growth

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
2014

ABSTRACT

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Abstract

This dissertation consists of three essays. In the first essay, “*The Asymmetric Cyclical Behavior of the U.S. Labor Market*,” I develop a search-and-matching model with endogenous job destruction and heterogeneous workers (in skill/productivity) that accounts for the asymmetry exhibited by cyclical fluctuations in the U.S. labor market and output while also generating (i) realistic volatility in unemployment and job-finding rates and (ii) preserving a downward-sloping Beveridge curve. The model delivers stark predictions for the time series of skill-specific unemployment rates that hold in the Current Population Survey (CPS) micro data once I sort workers by age and education. A general implication of the analysis is that the responsiveness of unemployment to stimulus policies increases substantially during recessions.

In the second essay, “*Volatility and Slow Technology Diffusion: The Case of Information Technologies*,” I address the following question: does business cycle volatility affect the rate at which new technologies are adopted? The answer to this question provides new insights on the link between volatility, total factor productivity (TFP), long-run economic growth, and cross-country differences in incomes per capita. The paper presents novel cross-country evidence on the link between volatility and time adoption lags. I find a highly statistically and economically significant negative relationship between volatility and the diffusion of three major information and communication technologies (ICT’s)—personal computers, internet and cell phones. Countries with more volatile growth rates of real GDP per capita

have higher time adoption lags. This negative relationship is rather robust and persists after controlling for cross-country differences in average growth rates of real GDP per capita. I also offer a simple stochastic model of technology adoption in which I derive in closed form the theoretical mapping between time adoption lags, growth, and volatility. In the model as in the data, there is a positive link between volatility and time adoption lags: the interaction of uncertainty with sunk costs of adoption generates a real option value of inaction which delays the adoption of new technologies with the consequent adverse effect on long-run economic growth.

In the third essay, “*Commodity Prices, Long-Run Growth and Fiscal Vulnerability*” (coauthored with Pietro Peretto), we study the short- and long-run effects of commodity price changes and how fiscal policy interacts with the amplification and propagation of external shocks to these prices. To this aim, we develop a Schumpeterian small open economy (SOE) model of endogenous growth that does not exhibit the scale effect. Because of the sterilization of the scale effect, commodity prices have level effects on economic activity but no steady-state growth effects. A general implication of our analysis is that the economy dynamic response to commodity price changes depends both on the structure of the tax code in place and on the policy response necessary to balance the government budget. We show that asset income taxation has negative steady-state growth effects. Furthermore, a positive tax rate on asset income acts as an automatic amplifier of external shocks to commodity prices and makes the effects of these shocks more persistent. Ultimately, our analysis provides insights on how to design welfare-enhancing tax policies for commodity-exporting countries.

To my father Francesco, my mother Antonietta, my siblings Davide and Federica,
and my beautiful little niece Nisia. Their unconditional love makes me happy.

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The Asymmetric Cyclical Behavior of the U.S. Labor Market

1.1 Introduction

Labor market fluctuations are large and strongly asymmetric. The U.S. employment rate contracts deeper and sharper during recessions than it expands in booms. Output, as employment, falls deeper below trend in recessions, but it declines as sharply as it raises.¹ These facts are well known, yet the literature lacks a quantitatively successful explanation.

Explaining these facts is interesting per se given the strong presence of asymmetry in U.S. macroeconomic series. Moreover, as I show in the paper, developing such an explanation leads to an explanation for the volatility of the U.S. labor market. In addition, understanding these nonlinearities is critical to address policy-relevant questions such as how the effectiveness of macroeconomic policies varies over the business cycle. By constructing a model in which asymmetries arise in equilibrium,

¹ See Mitchell (1913, 1927), Mitchell and Burns (1946), Neftci (1984), Sichel (1989, 1993), Long and Summers (1986), Falk (1986), Rothman (1991), McQueen and Thorley (1993), Verbrugge (1997), Belaire-Franch and Peiro (2003), Bai and Ng (2005), Hamilton (2005), and McKay and Reis (2008).

one can study state-dependent effects of stimulus policies more rigorously.

To inform my theoretical analysis, Section 1.2 details new asymmetry facts. First, I show that total hours worked feature cyclical asymmetries comparable to those of the U.S. employment rate. However, the asymmetric behavior is not exhibited in the fluctuations of hours per worker. These two facts suggest that the *extensive margin* is critical to understand the asymmetries of the labor input. Second, fluctuations in the U.S. participation rate (fraction of the population in the labor force) are symmetric. This fact suggests I can safely abstract from movements in and out of the labor force. Third, through a counterfactual exercise based on Shimer (2012), I document that both the job creation and job destruction margin of the labor market are needed to fully account for the asymmetric dynamics of the U.S. employment rate.

Motivated by the facts above, Section 3.2 develops a search-and-matching model of unemployment with endogenous job destruction and permanently heterogeneous workers (in skill/productivity) that accounts for the asymmetric fluctuations of the U.S. labor market and output.² The fundamental properties of the model are that recessions are initiated by a burst of job losses leading to a spike in unemployment followed by recoveries that are driven by a low aggregate job-finding rate.³ In what follows, I detail the key mechanism of the model that generates asymmetry in the labor market.

Consider first the scenario in which the economy rests at the steady state and it is hit by a positive shock to productivity. In this case, no endogenous separation occurs and job destruction is only due to exogenous separations. Therefore, unemployment dynamics are exclusively driven by a high aggregate job-finding rate. In this case, the distribution of skills in the unemployment pool closely replicates the distribution

² The permanent nature of heterogeneity is reasonable in the sense that over the cycle there is not much workers can do to change their skills/productivity.

³ See Davis et al. (2006, 2010), Fujita and Ramey (2009), and Elsby et al. (2009) for evidence supporting this view.

of skills in the economy. Consider now a negative shock to productivity. Given the heterogeneity in workers' skill/productivity, the model features a “reservation property” such that all matches with workers with productivity below a cutoff value are endogenously destroyed. Since laid-off workers are permanently low-skilled, they are not rehired until aggregate productivity returns to its normal level. Thus, in this case the unemployment pool is characterized by a distribution of skills that is skewed to the left. It is this change in the distribution of skills of the unemployment pool over the cycle that generates asymmetries in the aggregate job-finding rate.

In principle, worker heterogeneity aside, endogenous job destruction alone can give rise to “*spiky*” dynamics in the job-separation rate leading to asymmetries in employment. For example, consider the Diamond-Mortensen-Pissarides (DMP) model augmented with shocks to the separation rate as in Shimer (2005) or the endogenous job separation model à la Mortensen and Pissarides (1994). While these models could (at least qualitatively) account for the asymmetries in the U.S. data, it is well known that this class of models generate a counterfactual Beveridge curve and fail to generate realistic volatility in vacancies and job-finding rates. Furthermore, as discussed above, I document in the paper that the asymmetries in the U.S. data are due to both job-separation and job-finding rates. As such a model relying exclusively on endogenous job destruction would be counterfactual vis-à-vis this observation. Thus, while relying solely on endogenous job destruction would “match” the asymmetry facts it will be at odds with other key empirical facts. It is the interaction between the endogenous separation margin and the permanent heterogeneity that allows the model to account for the asymmetry facts jointly with (i) realistic volatility in unemployment and job-finding rates, as well as (ii) the correct relative contribution of job-separation and job-finding rates to the asymmetry properties of the data.

In Section 1.4, I calibrate the model and evaluate its quantitative implications. I

argue the model is able to replicate the key asymmetry facts as well as generate realistic volatility in unemployment and job-finding rates while preserving a downward-sloping Beveridge curve (unemployment-vacancy locus). Specifically, it is well known that accounting for the volatility of the U.S. labor market is one of the puzzles in analysis based on DMP models.⁴ Crucially, the model does not rely on a high calibration of the worker's outside option. Precisely, worker heterogeneity jointly with a fixed outside option delivers a spectrum of replacement ratios (i.e., worker's outside option as percent of the wage). This leads to a disconnect between average and marginal workers. On the one hand, marginal workers are the least productive in the labor force and the ones laid off during recessions. These workers feature high replacement ratios but account for only 5 percent of the labor force. On the other hand, high-skilled workers have low replacement ratios and account for the bulk of the workforce. As such the economy features *on average* a low replacement ratio.

As a by-product, the model provides stark predictions for the time series of skill-specific unemployment rates. In Section 1.5, I use CPS micro data for the period 1976:M1- 2013:M2 to test these predictions. Specifically, the model predicts that less productive workers account for the bulk of the average and variation over time of the unemployment rate. Since age and education are natural proxies for skills, I analyze their unemployment behavior and indeed find that (i) young and least-educated workers experience average unemployment rates that are up to nine times that of prime-aged workers and (ii) they account for approximately 70 percent of the time series variation in the U.S. unemployment rate. These facts provide strong support for the main prediction of the model: understanding cyclical movements of low-skilled workers is critical to explain the large fluctuations of the U.S. labor market.

The model has a wide range of implications for the design of macroeconomic

⁴ See Andolfatto (1996), Shimer (2005), and Costain and Reiter (2008).

policies. A general prediction of the analysis is that the effectiveness of stimulus policies varies over the business cycle. In Section 1.6, I show that the effects of policies that restore the profitability of low-productivity matches are time varying: these policies are much more effective during economic downturns than expansions. That is, the economy features impulse responses that vary with the state of the economy.

This paper relates to the literature on asymmetric cycles. Most of this literature consists of papers focusing on output and/or investment.⁵ Two exceptions are the studies on the labor market of Andolfatto (1997) and McKay and Reis (2008). However, this paper is the first attempt to provide a unified and quantitatively successful explanation for the volatility and asymmetry of the U.S. labor market. Importantly, I also show how cyclical asymmetries connect with nonlinearities in the amplification and propagation of shocks and argue about their relevance for policy analysis.

1.2 Asymmetry Facts

This section details the basic facts on hours, unemployment and output that motivate the paper and discusses new facts on their asymmetry properties. I consider the employment rate (fraction of the labor force working in a given month, one minus the unemployment rate) as the main cyclical indicator of the U.S. labor market. Following Sichel (1993), I measure deepness and steepness asymmetry in an economic time series with the skewness coefficient of respectively its detrended counterpart—*asymmetry in levels*—and log-first-differences—*asymmetry in growth rates*. To test for asymmetry against the null hypothesis of symmetry, I use the test developed by Bai and Ng (2005). To isolate fluctuations at business cycle frequencies, I detrend the data with the Hodrick-Prescott (HP) filter. Figure 1.1 summarizes the main

⁵ See Ball and Mankiw (1994), Acemoglu and Scott (1997), Kocherlakota (2000), Hansen and Prescott (2005), Jovanovic (2006), Van Nieuwerburgh and Veldkamp (2006), Devereux and Siu (2007), Cheremukhin and Tutino (2013), and Görtz and Tsoukalas (2013).

asymmetry facts for employment and output.

1.2.1 Hours, unemployment, and output

In Figure 1.2, I report percent deviations from the HP trend for the U.S. employment rate for the period 1948:Q1-2012:Q2. This figure illustrates that the largest deviations below trend (in absolute value) exceed the largest deviations above trend, i.e., troughs are deeper than peaks are tall. For example, there are six NBER-dated recessions during which the U.S. employment rate falls 2 percent below trend or more. On the other hand, the employment rate barely reaches as high as 2 percent above trend over the same period. This asymmetry between peaks and troughs indicates deepness in employment cycles. Moreover, the rate at which the employment rate falls during downturns exceeds the rate at which it raises during upturns. This asymmetry in rates of change over the contraction and expansion phases indicates steepness in employment.

Fact 1. *Employment rates display negative skewness in levels and growth rates.*

Fact 1 is well-known in the empirical literature on asymmetric cycles. Since Sichel (1993), this fact has been confirmed by many authors.⁶ Panel A and B in Figure 1.1 show that the empirical distributions of respectively detrended and first-difference employment rates are left-skewed. For the detrended series, the skewness coefficient equals -0.581 and it is highly statistically significant with a p-value of 0.005 . Negative skewness in the detrended series is capturing the fact there are a relatively large number of small deviations above trend compared to a relatively small number of large deviations below trend. The tails of the distribution reflects the asymmetry between peaks and troughs. For the series in first differences, the skewness coefficient equals -1.115 with a p-value of 0.001 . The skewness coefficient

⁶ See Verbrugge (1997) and Bai and Ng (2005) among others.

of growth rates is a simple statistics apt to identify *large* and *sudden* changes in employment rates. Negative skewness captures the presence of a relatively large number of small positive changes compared to a small number of large negative changes. These large negative changes in the employment rate occur at the onset of U.S. recessions.

To strengthen Fact 1, I further document that cyclical asymmetries also characterize age-, gender- and education-specific employment rates and are a robust feature across U.S. states and different sectors of the U.S. economy. These findings prove that cyclical asymmetries characterize the entire labor market as such they can be meaningfully studied as an aggregate phenomenon (see Appendix B.1 for further details).

Furthermore, I show that total hours worked feature cyclical asymmetries comparable to those of the U.S. employment rate. However, the asymmetric behavior is not present in hours per worker. These two facts suggest that the *extensive margin* of the labor market is critical to understand the asymmetries of the labor input. This observation is reminiscent of the well-known fact that most of the volatility of hours worked at business cycle frequencies is due to fluctuations in the number of employed as opposed to fluctuations in hours per employed worker.⁷ Table 1.1 shows results for hours and hours per worker.

There is some debate in the literature as whether the employment rate (as fraction of the labor force) or the employment-population ratio is a better indicator representing the state of the labor market. For example, Blanchard et al. (1990) argue that the number of workers moving directly into employment from out-of-the-labor force is as large as the number who move from unemployment to employment.⁸

⁷ See Rogerson and Shimer (2011) among others.

⁸ See also Flinn and Heckman (1983), Juhn et al. (1991), Jones and Riddell (1999) and Cole and Rogerson (2001) for further discussions on whether the categories “unemployed” and “out-of-the-labor force” are different labor force states.

To address this issue, I study the asymmetry properties of the U.S. participation rate, i.e., fraction of the population in the labor force (employed plus unemployed to population ratio). Figure 1.3 clearly shows that the null hypothesis of symmetry cannot be rejected in the data.

The results in Figure 1.3 establish that fluctuations in the U.S. participation rate are symmetric. This fact suggests that to understand cyclical asymmetries in employment, I can abstract from movements in and out of the labor force.

In Figure 1.1, Panel C and D show the empirical distributions of detrended and first-difference industrial production (IP). In Panel C, the skewness coefficient for the detrended series equals -0.644 and is highly statistically significant with a p-value of 0.001. However, we cannot reject the null hypothesis of symmetry for the series in first differences. Panel D reports a skewness coefficient of -0.238 with a p-value of 0.258.

Fact 2. *Output displays negative skewness in levels but no skewness in growth rates.*

Fact 2 is consistent with the evidence documented by Sichel (1993), Falk (1986), Long and Summers (1986) and more recently McKay and Reis (2008). Appendix B.1 reports further results showing that steepness asymmetry is rejected in the data for several measure of real output as real GDP, business sector and nonfarm business sector output.

1.2.2 Inflows and Outflows of Unemployment

In this section, I document new facts about the asymmetric dynamics of the U.S. labor market. I compute counterfactual employment rate series based on Shimer (2012) and assess the relative contribution of job-finding and job-separation rates to the asymmetry properties of the U.S. employment rate.

Job-Finding and Separation Rates

In the standard DMP framework, the continuous time law of motion for employment is the following differential equation:

$$\dot{e}(t) = u(t)f(t) - e(t)s(t), \quad (1.1)$$

where a dot denotes a time derivative, $u(t)$ denotes the number of unemployed, $f(t)$ and $s(t)$ denote respectively job-finding rate (JFR) and job-separation rate (JSR). To estimate $f(t)$ and $s(t)$, I follow Shimer (2012). As such, I simply summarize here, and refer the reader to Shimer's work for further details.⁹ The approach uses monthly data on employment, unemployment and short-term unemployment from the CPS of BLS. All variables are in level (thousands of persons) and short-term unemployment refers to the number of unemployed persons for less than five weeks. The job-finding probability, F_t , between month t and $t+1$ can be computed as:

$$F_t = 1 - \frac{u_{t+1} - u_{t+1}^s}{u_t},$$

where u_t is the unemployment level in month t and u_{t+1}^s , short-term unemployment level, is the number of persons unemployed for less than 5 weeks in month $t+1$. The job-finding rate is then $f_t \equiv -\log(1 - F_t) \geq 0$. Shimer (2012) shows that one can solve the differential equation (1.1) forward to obtain an implicit nonlinear expression for the job-separation rate s_t . Given the job-finding rate f_t , data on employment and unemployment levels then the job-separation rate s_t is uniquely determined. Finally, I calculate quarterly job-finding and job-separation rates by averaging over the corresponding monthly observations.

⁹ The job-finding and job-separation rates are derived under two assumptions: 1) workers do not transit in and out of the labor force; 2) workers are homogeneous with respect to job-finding and job-separation probabilities.

Employment Counterfactuals

To compute employment counterfactuals, I follow Shimer (2012) by approximating the U.S. employment rate series using its theoretical steady-state value from equation (1.1), associated with the contemporaneous job-separation and job-finding rate—*Stochastic equilibrium*:

$$e_t^{\text{se}} = \frac{f_t}{s_t + f_t}. \quad (1.2)$$

Hall (005b) and Shimer (2012) show that the stochastic equilibrium is a strikingly good approximation for the actual U.S. employment rate (see Appendix B.1 for a plot of actual and counterfactual employment). Given equation (1.2), I construct two counterfactual series. The first is an employment rate series that only allows for variation in the job-finding rate:

$$e_t^{\text{jfr}} = \frac{f_t}{\bar{s} + f_t}, \quad (1.3)$$

where \bar{s} is the sample average of the job-separation rate s_t . The second counterfactual, instead, only allows for variation in the job-separation rate:

$$e_t^{\text{jsr}} = \frac{\bar{f}}{s_t + \bar{f}}, \quad (1.4)$$

where \bar{f} is the sample average of the job-finding rate f_t .

Which counterfactual series better accounts for the asymmetric dynamics of the U.S. employment rate? To answer this question, I implement the asymmetry tests of Section 1.2.1 on the counterfactual employment rates e_t^{jfr} and e_t^{jsr} .¹⁰

¹⁰ The counterfactual employment series (1.3) and (1.4) are constructed under the assumption that job-finding and job-separation rates are two independent sources of fluctuations. The independence assumption holds in all DMP models with exogenous and constant job-separation rate and in the Mortensen and Pissarides (1994)'s model with endogenous job separations.

In Table 1.2, the first row shows that both counterfactual employment series, e^{jfr} and e^{jsr} , display significant negative skewness as in the U.S. data. I conclude that both job-finding and job-separation rates are critical in accounting for the depth of fluctuations in the U.S. employment rate.

Fact 3. *Job-finding and job-separation rates are jointly responsible for the negative skewness in levels in the employment rate.*

In Table 1.2, the second row forcefully shows that negative steepness in the U.S. employment rate is entirely driven by the dynamics of job-separation rates.

Fact 4. *Job-separation rates are the only responsible for the negative skewness in growth rates in the employment rate.*

Barnichon (2012) reaches the same conclusion in Fact 4 using a different approach. Furthermore, notice that if the dynamics of the U.S. employment rate was only driven by job-finding rates then we would observe positive skewness in the distributions of employment rate changes, i.e., positive instead of negative steepness as in the U.S. data.

Overall, this section suggests that both job-finding and job-separation rates are relevant to fully account for the asymmetric dynamics of the U.S. labor market. Specifically, job-separation rates are the only responsible for the sharp reductions in the U.S. employment rate that occur at the onset of recessions. Job-finding rates are instead mostly responsible for why U.S. employment spends more time below trend. The combined of Fact 3 and 4 contribute to the ongoing debate on the proper treatment of the separation margin over which there is still little consensus in the literature. This debate has been recently revived by a series of papers with contrasting conclusions. On the one hand, Hall (005a,b) and Shimer (2012) attribute most of the volatility in unemployment at business cycle frequencies to the dynamics of job-

finding rates—“hiring-driven view.” On the other hand, Fujita and Ramey (2009) and Elsby et al. (2009) resurrect the role played by the job-separation margin. While these papers focus on second-order moments of the data, I analyze their asymmetry properties.

1.3 Model

In this section, I extend the textbook Pissarides (2000)’s model by introducing worker heterogeneity. Precisely, workers exhibit permanent differences in skill/productivity while employers are identical.

1.3.1 *Economic Environment*

Time is discrete and indexed by $t \in \{0, 1, \dots, \infty\}$. Workers are heterogeneous in their skills. Specifically, I consider an economy populated by M types of workers indexed by $x \in \{x_1 < \dots < x_M\}$. The type x is a *permanent* characteristic of the worker which is perfectly observable to employers. I ex-ante sort workers in submarkets based on their types. Therefore, the aggregate labor market is organized in M submarkets indexed by worker’s type x . In each submarket, there is a unit mass of infinitely lived workers of type x , either employed, $e_t(x) \in [0, 1]$, or unemployed and searching for a job, $u_t(x) \in [0, 1]$. The aggregate labor force is then $\sum_x (e_t(x) + u_t(x)) = M$. Each worker is endowed with an indivisible unit of labor. I adopt a 2-state representation of the labor market such that I abstract from movements in and out of the labor force. There is no on-the-job search, therefore only unemployed workers can search for a job. The economy is also populated by a continuum of identical and infinitely lived employers, either producing output, $y_t(x)$, or posting job vacancies, $v_t(x)$, to hire unemployed workers of type x . Workers and employers have risk-neutral preferences and discount future payoffs at rate $\beta \in (0, 1)$. I assume workers and employers can respectively search for jobs and post vacancies only in one submarket at the

time. Specifically, employers are allowed to optimally choose how many vacancies to create and in which submarket to locate them. Workers instead do not move across submarkets.¹¹

Matching and production technologies. I adopt the standard view of matching frictions in the labor market and postulate the existence of a matching technology.

Assumption 1 (Matching function). *In each submarket x , the matching function $m(v_t(x), u_t(x)) = \mu v_t(x)^\alpha u_t(x)^{1-\alpha}$ is strictly increasing and concave in both arguments and homogeneous of degree one in the number of unemployed workers, $u_t(x)$, and vacancies, $v_t(x)$. The scale parameter μ measures matching efficiency, and $\alpha \in (0, 1)$ is the elasticity of new matches, or hires, $m(v_t(x), u_t(x))$ with respect to the number of vacancies $v_t(x)$.*

Each submarket is characterized by the tightness ratio $\theta_t(x) \equiv v_t(x)/u_t(x)$.¹² An unemployed worker of type x finds a job with probability $\phi(\theta_t(x)) = \mu\theta_t(x)^\alpha$ that is strictly increasing and concave in $\theta_t(x)$, i.e., $\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$, and a vacancy posted in submarket x is filled with probability $\rho(\theta_t(x)) = \phi(\theta_t(x))/\theta_t(x) = \mu\theta_t(x)^{\alpha-1}$ that is strictly decreasing and convex in $\theta_t(x)$, i.e., $\rho'(\cdot) < 0$ and $\rho''(\cdot) > 0$. I assume workers and firms that are matched at time t produce output at time $t + 1$. A worker-employer match produces output via a linear technology.

Assumption 2 (Production function). *In each submarket x , output is produced according to the linear production function, $y_t(x) = z_t x$, where z_t is the aggregate stochastic component of labor productivity and x is a time-invariant worker-specific component.*

¹¹ This modelling choice is without loss of generality. One can show that in the equilibrium under directed search any active submarket is visited exclusively by one type of worker. Hence, the labor market is endogenously segmented by worker's type. See Menzio and Shi (2010) and Carrillo-Tudela and Visschers (2013) for a similar result.

¹² Petrongolo and Pissarides (2001) provide evidence supporting the constant-returns-to-scale assumption.

Shocks. Fluctuations are driven by exogenous variations in the aggregate component of labor productivity $\{z_t\}_{t=0}^{\infty}$.¹³

Assumption 3 (Symmetric shocks). *The stochastic process for the exogenous state $\{z_t\}$ is an asymptotically stationary \mathcal{S} -state Markov chain $(\mathcal{Z}, \Pi, \pi_0)$, with a unique, symmetric, and uni-modal stationary distribution π_{∞} .*¹⁴

1.3.2 Equilibrium Characterization

I next characterize the equilibrium dynamics of a single submarket $x \in \{x_1, \dots, x_M\}$. Abusing notation slightly, let $\phi_s(x) \equiv \phi(\theta_s(x))$ and $\rho_s(x) \equiv \rho(\theta_s(x))$ denote respectively the job-finding and job-filling rate in submarket x when the random state of the model economy is $s \in \{1, \dots, \mathcal{S}\}$.

Employer's problem. The employer decides either to remain in the match, and get the value $J_s^c(x)$, or to post a job vacancy, and get the value $V_s(x)$. Specifically, let $J_s(x)$ denote the value for an employer in submarket x if the economy is in state s ,

$$J_s(x) = \max \{J_s^c(x), V_s(x)\}$$

with

¹³ Empirical evidence by Abraham and Katz (1986) and Blanchard et al. (1989) suggests that recessions are driven by aggregate activity shocks. In line with this evidence, I assume that fluctuations are driven by aggregate disturbances.

¹⁴ The state space for $\{z_t\}$ is the finite set $\mathcal{Z} = \{z_1, \dots, z_{\mathcal{S}}\}$. The states $z_s \in \mathcal{Z}$ take on \mathcal{S} possible values, $z_1 < \dots < z_{s_m} < \dots < z_{\mathcal{S}}$, that are symmetrically spaced around the median state z_{s_m} which I normalize to one. The probability transition matrix is a $(\mathcal{S} \times \mathcal{S})$ matrix $\Pi = [\pi_{s,s'}]$ with transition probabilities $\pi_{s,s'} = \text{Prob}\{z_{t+1} = z_{s'} | z_t = z_s\}$, non-negative and stochastic, i.e., $\pi_{s,s'} \geq 0$ and $\sum_{s'} \pi_{s,s'} = 1$, for all (s, s') . The stationary distribution $\pi'_{\infty} = \lim_{t \rightarrow \infty} \pi'_0 \Pi^t$ is symmetric and uni-modal, satisfying: $\pi_{\infty}^s = \pi_{\infty}^{S-s+1}$ and $\pi_{\infty}^s < \pi_{\infty}^{s+1}$, for $s \in \{1, \dots, s_m - 1\}$, where π_{∞}^s denotes the s -th element of the probability vector π_{∞} . The probability distribution at time $t = 0$ is the $(\mathcal{S} \times 1)$ vector π_0 .

$$J_s^c(x) = p_s(x) - \omega_s(x) + \beta \sum_{s'} \pi_{s,s'} \left\{ [1 - \delta_{s'}(x)] J_{s'}(x) + \delta_{s'}(x) \max_x (V_{s'}(x), 0) \right\},$$

where $J_s^c(x)$ is the value of remaining in the match. $\Pi_s(x) = z_s x - \omega_s(x)$ are profits accruing to the employer, $p_s(x) \equiv z_s x$ denotes output, $\omega_s(x)$ is the wage payment to the worker, and $\delta_s(x)$ is the state-contingent job-separation rate. The price of output is normalized to one. The economy transits from state s to the next period state s' according to the transition probability $\pi_{s,s'}$. The value for the employer to post a vacancy in submarket x if the economy is in state s is,

$$V_s(x) = -k(x) + \beta \sum_{s'} \pi_{s,s'} \left\{ \rho_s(x) J_{s'}(x) + [1 - \rho_s(x)] \max_x (V_{s'}(x), 0) \right\}, \quad (1.5)$$

where $k(x)$ is the unit cost to keep a job vacancy open for one period.

Worker's problem. The worker decides either to remain in the match, and get the value $W_s^c(x)$, or to be unemployed, and get the value $U_s(x)$. Specifically, let $W_s(x)$ denote the value for a worker in submarket x if the economy is in state s ,

$$W_s(x) = \max \{W_s^c(x), U_s(x)\}$$

with

$$W_s^c(x) = \omega_s(x) + \beta \sum_{s'} \pi_{s,s'} \left\{ [1 - \delta_{s'}(x)] W_{s'}(x) + \delta_{s'}(x) U_{s'}(x) \right\},$$

where $W_s^c(x)$ is the value of working in a continuing match. The value for the worker of being unemployed and searching for a job in submarket x when the economy is in state s is,

$$U_s(x) = \lambda + \beta \sum_{s'} \pi_{s,s'} \left\{ \phi_s(x) W_{s'}(x) + [1 - \phi_s(x)] U_{s'}(x) \right\}, \quad (1.6)$$

where λ is the worker's outside option, i.e., income earned when unemployed.

Timing. At time t , the aggregate shock z_t realizes. Endogenous separations (layoffs) take place. The search and matching process follows: employers post job vacancies on one side and unemployed workers search for jobs on the other side. Unemployed workers matched with employers at time t become productive at time $t + 1$. Finally, production takes place.

Nash-bargaining equilibrium. Upon matching, the employer and worker enter Nash bargaining to determine the wage. The tightness ratio $\theta_s(x)$ is taken parametrically by agents and determined in equilibrium by the collection of all the individual optimal allocations.

Definition 4 (Nash-bargaining equilibrium). *A Nash-Bargaining Equilibrium is a collection of value functions $J_s(x)$ and $V_s(x)$ for the employers, $U_s(x)$ and $W_s(x)$ for the workers, wage payments $\omega_s(x)$, and tightness ratios $\theta_s(x)$, such that for each $s \in \{1, \dots, \mathcal{S}\}$,*

1. *Employers are optimizing, taking as given the tightness ratios $\theta_s(x)$ and the wage payments $\omega_s(x)$. That is, employers with a filled job prefer to remain matched with the worker rather than posting a vacancy, $J_s(x) - V_s(x) > 0$;*
2. *Workers are optimizing, taking as given the tightness ratios $\theta_s(x)$ and the wage payments $\omega_s(x)$. That is, workers in a job prefer to remain matched with an employer rather than being unemployed, $W_s(x) - U_s(x) > 0$;*
3. *The free-entry condition is satisfied, $V_s(x) = 0$;*

4. *Wage payments for newly formed and continuing matches solve the generalized Nash-bargaining problem:*

$$\omega_s(x) = \arg \max [W_s(x) - U_s(x)]^\eta \cdot [J_s(x) - V_s(x)]^{1-\eta},$$

where η denotes the bargaining weight of workers.

The Nash-bargaining solution implies that worker and employer receive a constant and proportional share of the total surplus, $W_s(x) - U_s(x) = \eta S_s(x)$ and $J_s(x) = (1 - \eta)S_s(x)$, where $S_s(x) \equiv W_s(x) + J_s(x) - U_s(x)$. The wage payment to the worker is,

$$\omega_s(x) = (1 - \eta) \lambda + \eta [z_s x + k(x) \theta_s(x)]. \quad (1.7)$$

Under Nash-bargaining, the equilibrium dynamics of the model economy is fully characterized by the Bellman value equation for total match surplus,

$$S_s(x) = \max \{S_s^c(x), 0\} \quad (1.8)$$

with

$$S_s^c(x) = z_s x - \lambda + \beta \sum_{s'} \pi_{s,s'} [1 - \delta_{s'}(x) - \eta \phi_s(x)] S_{s'}(x),$$

where $S_s^c(x)$ is the continuation value of the total match surplus. After imposing $V_s(x) = 0$ for all $s \in \{1, \dots, \mathcal{S}\}$, equation (1.5) becomes,

$$k(x) = \beta \rho_s(x) \sum_{s'} \pi_{s,s'} (1 - \eta) S_{s'}(x). \quad (1.9)$$

Equation (1.9) captures a central aspect of the model dynamics. Employers post vacancies up to the point where the expected surplus from making a match,

$\beta \sum_{s'} \pi_{s,s'}(1 - \eta)S_{s'}(x)$, is exactly offset by the expected recruiting costs, $k(x)/\rho_s(x)$. As the employers post more vacancies, the tightness ratio $\theta_s(x)$ rises, the probability to fill the posted vacancy $\rho_s(x)$ decreases, and the point of zero net expected surplus is achieved. This mechanism pins down the key variable of the model, that is, the vacancy-unemployment ratio, $\theta_s(x)$. In a tight market with a relatively high ratio of vacancies to unemployment, it is easy for job seekers to find jobs—the job-finding rate $\phi(\theta_s(x))$ is high—and difficult for firms to hire—the job-filling rate $\rho(\theta_s(x))$ is low.

1.3.3 Endogenous Job Destruction

Equation (1.8) determines when jobs are endogenously destroyed—*Job destruction margin*. Since the continuation value of the total match surplus $S_s^c(x)$ is monotonically increasing in labor productivity, $p_s(x) = z_s x$, job destruction satisfies the reservation property. There exists a unique cutoff value for the aggregate state, $\bar{z}(x)$, such that all matches with workers of type x are endogenously destroyed when hit by an adverse aggregate shock, $z_s \leq \bar{z}(x)$. Since employers and workers have the option to separate at no cost, a match continues in operation for as long as its value is above zero. Note that under Nash bargaining, separations are bilaterally efficient in that employers and workers agree on the decision to destroy existing matches. Hence, large negative shocks induce job destruction but the choice of when to destroy the job is optimally chosen by employers and workers, jointly. Therefore, the job destruction rate $\delta_s(x)$ is the following step function,

$$\delta_s(x) = \begin{cases} \delta & \text{if } z_s > \bar{z}(x) \Leftrightarrow S_s^c(x) > 0 \\ 1 & \text{if } z_s \leq \bar{z}(x) \Leftrightarrow S_s^c(x) \leq 0 \end{cases} \quad (1.10)$$

for each submarket $x \in \{x_1, \dots, x_M\}$. Since total match surpluses are increasing in x , the cutoff on the aggregate state varies across workers' types, $\bar{z}(x_1) > \dots > \bar{z}(x_M)$.

This implies that matches with low-skilled workers are more likely to be destroyed, i.e., low-skilled workers face an higher probability to be laid off. Notice that the reservation property defines three regions in the productivity space: (1) $S_s^c(\underline{x}) = 0$. Matches with workers of type $x \leq \underline{x}$ are never active in that total match surpluses are negative for any realization of the aggregate shock; (2) $S_1^c(\bar{x}) = 0$. Matches with workers of type $x > \bar{x}$ feature an exogenous constant rate of job destruction in that total match surpluses are always above zero; and (3) matches with workers of type $\underline{x} < x \leq \bar{x}$ feature both endogenous and exogenous rates of job destruction. This happens because exogenous variations in the aggregate state lead swings in total match surpluses that occasionally hit the non-negativity constraint, $S_s^c(x) \geq 0$. Hence, heterogeneous jobs respond differently to common shocks. Specifically, low-skilled workers experience more frequent and longer unemployment spells. Davis (2005) argues that layoffs are associated with greater unemployment incidence and longer unemployment spells than quits. To summarize, endogenous job destruction operates through a “selection mechanism.” As the model economy is hit by an adverse aggregate shock, low-skilled workers are laid off. High-skilled workers instead enter unemployment spells at a constant exogenous rate. This implies that the aggregate separation rate spikes at the onset of economic downturns. Fujita and Ramey (2009) and Elsby et al. (2009) provide evidence supporting this prediction. The model also predicts that layoffs are countercyclical. As such, a disproportionate part of unemployment inflows during a downturns consists of laid-off workers. Davis et al. (1998) and Elsby et al. (2013) provide evidence supporting this view. Furthermore, since in the model laid-off workers are low-skilled, during downturns low-skill workers are over-represented in the group flowing into unemployment which in turn leads to a decrease in the average skill level of the unemployed pool, i.e., $\sum_x x u_t(x) / \sum_x u_t(x)$ falls during downturns.

To further sharpen intuition, I next focus on the labor market in steady state,

i.e., $z_t = z_s$ for $s \in \{1, \dots, \mathcal{S}\}$ and all t . This enables me to derive the cutoff value on the aggregate state analytically. To this aim, after manipulating equation (1.8), one gets

$$\hat{p}(x) \equiv \hat{z}x = \lambda. \quad (1.11)$$

Equation (1.11) is a key condition of the model. The left hand side of the equation is the lowest productivity acceptable to employers with a filled job for remaining matched with workers of type x rather than dissolve the match. The right hand side is instead the opportunity cost of employment for workers of type x , which consists of the worker's outside option λ . It is easy to verify that when x decreases the cutoff on the aggregate state \hat{z} needs to increase for (1.11) to hold, i.e., the cutoff value $\hat{z}(x) = \lambda/x$ is decreasing in worker's type x . In other words, high-skilled workers are laid off at lower realizations of the aggregate shock. Furthermore, for given x , an increase in λ requires an equal increase in $\hat{z}(x)$ for (1.11) to hold. This implies that workers of type x are more likely to be laid off and that a larger fraction of the labor force is now at risk of layoffs. The analysis suggests that workers' outside options are critical to understand the destruction margin of the labor market. This is particularly true for low-skilled workers who are always at the margin between participating the labor market and enjoying the value of non-market activities. A direct implication of this argument is that exogenous increases in workers' outside options, i.e., changes in the relative return to market versus non-market activity, lead to longer unemployment durations for low-skilled workers and extend the endogenous destruction region to workers with higher skills.

1.3.4 Job Creation and Job Rationing

In the model, endogenous job destruction and job creation are entwined by the reservation productivity in equation (1.11) that determines when (i) matches with

low-skilled workers are endogenously destroyed and when (ii) low-skilled workers previously laid-off are viable for hiring (in the sense that they generate a positive surplus). This implies that the (permanent) heterogeneity in workers' skills that matters for the endogenous separation decision also affects hiring decisions. Specifically, it becomes profitable to hire low-skilled workers only when the aggregate state returns to a level that is high enough to guarantee positive surpluses. This is the essence of the selection mechanism that drives the dynamics of the model during downturns and recoveries. The model predicts that jobs are rationed during recessions.

Definition 5 (Job rationing). *Following Michaillat (2012), I define job rationing as the situation in which positive unemployment would persist even if the recruiting cost $k(x)$ was zero.*

To understand how job rationing emerges in equilibrium, notice that the threshold $\hat{p}(x)$ in equation (1.11) is independent of the cost of posting a vacancy $k(x)$. Employers would find unprofitable to hire low-skilled workers with $p_s(x) \leq \hat{p}(x)$ even if the cost to post a vacancy was zero. This is intuitive since skills are a time-invariant characteristic of the labor force and low-skilled workers previously laid off remain low-skilled at the time of hiring.

To further sharpen intuition, I focus on the labor market in steady state, i.e., $z_t = z_s$ for $s \in \{1, \dots, \mathcal{S}\}$ and all t . This enables me to represent the equilibrium diagrammatically. To this aim, consider steady-state employment in submarket x ,

$$e_s(x) = \frac{\phi(\theta_s(x))}{\delta_s(x) + \phi(\theta_s(x))}. \quad (1.12)$$

After manipulating equations (1.8) and (1.9), one gets

$$\underbrace{z_s x - \lambda}_{\text{Net marginal profits}} = \underbrace{\frac{k(x)}{\beta(1-\eta)} \cdot \left[\frac{1 - \beta(1 - \delta_s(x))}{\rho(\theta_s(x))} + \eta\beta\theta_s(x) \right]}_{\text{Marginal recruiting expenses}}. \quad (1.13)$$

Equations (1.12) and (1.13) fully characterize the steady-state equilibrium. Figure 1.4 represents the equilibrium diagrammatically.

In Figure 1.4, Panel A shows the equilibrium in the submarket for high-skilled workers, i.e., workers of type $x_{\text{high}} > \bar{x}$. As discussed in Section 1.3.3, these workers never experience endogenous separations in equilibrium, i.e., $S_s(x_{\text{high}}) > 0$ for all realization of the aggregate shock. Equilibrium employment is obtained at the intersection of net marginal profits and recruiting expenses curves. Panel B depicts the equilibrium for high-skilled workers as I progressively decrease the cost of posting a job vacancy $k(x_{\text{high}})$. A decrease in the cost of posting a vacancy makes the marginal recruiting expenses curve shift rightward which in turn leads to an increase in employment. This mechanism is the cornerstone of the frictional view of labor markets. As the recruiting cost goes to zero, unemployment tends to vanish with high-skilled workers approaching full employment. Panel C shows how equilibrium employment changes as the aggregate state falls. The fall in the aggregate state makes the net marginal profits curve shift downwards, which in turn leads to a decrease in employment. An adverse shock to profits curtails the incentives to vacancy posting which in turn leads a drop in the market tightness ratio, job-finding probabilities and employment. This is the core mechanism driving recessionary unemployment in search-and-matching models. For high-skilled workers all unemployment is frictional at any point of the business cycle. Panel D depicts instead the equilibrium for low-skilled workers, i.e., workers of type $\underline{x} < x_{\text{low}} \leq \bar{x}$, when the adverse aggregate shock makes their productivity hit the rationing threshold, i.e., $p_s(x_{\text{low}}) \leq \hat{p}(x)$. For these

workers, employment goes to zero irrespective of the cost of posting a vacancy.

The model predicts that during downturns job-finding probabilities endogenously fall for all workers, though disproportionately more for low-skilled workers previously laid-off. This happens because jobs are rationed. At each point in time, aggregate unemployment consists of frictional (U_t^F) and rationing (U_t^R) unemployment,

$$U_t = \underbrace{\sum_{p_t(x) \leq \hat{p}(x)} u_t(x)}_{\substack{\text{Rationing} \\ \text{unemployment}}} + \underbrace{\sum_{p_t(x) > \hat{p}(x)} u_t(x)}_{\substack{\text{Frictional} \\ \text{unemployment}}} = U_t^R + U_t^F. \quad (1.14)$$

Furthermore, the relative importance of each component depends on the magnitude of the adverse aggregate shock. Specifically, deeper downturns are characterized by larger shares of rationing unemployment, i.e., $u_t^R \equiv U_t^R / (U_t^R + U_t^F)$ is decreasing in the aggregate shock. This happens because a bigger fraction of the aggregate labor force hits the rationing threshold, $\hat{p}(x)$, in equation (1.11).

Absence of Job Rationing in Existing Search-and-Matching Models with Endogenous Separations

Michaillat (2012) discusses the absence of job rationing in standard search-and-matching models with a constant exogenous rate of job destruction. As such, I refer the reader to Michaillat's work for further details. In this section, I show that job rationing is also absent in models with endogenous separations à la Mortensen and Pissarides (1994). To this aim, I focus on a discrete time version of Mortensen and Pissarides's model as discussed in Fujita and Ramey (2012). In this setting, workers are identical. Heterogeneity arises ex-post due to match-specific idiosyncratic shocks to productivity. The match-specific component of productivity, $x \in \{x_1, \dots, x_M\}$, switches to a new value with probability ξ . In this latter event, the value of x is drawn randomly according to the c.d.f. $G(x)$. Matches are exogenously destroyed at

the constant rate δ and those whose productivity is below a cutoff are endogenously destroyed. All new matches start at $x = x_M$. Hence, workers are homogeneous at the time of hiring. I refer the reader to Fujita and Ramey's work for further details on the model's structure.

To study the equilibrium theoretically, I focus on steady states (i.e., $z_t = z$ for all t).

Proposition 6 (Job rationing in models à la Mortensen and Pissarides).

Under assumptions in Mortensen and Pissarides (1994), $\lim_{k \searrow 0} \theta(k) = +\infty$ and $\lim_{k \searrow 0} e(k) = 1$, i.e., full employment, or $\lim_{k \searrow 0} \theta(k) = 0$ and $\lim_{k \searrow 0} e(k) = 0$, i.e., the entire labor market shuts down.

Proof. See Appendix A.3. ■

Hence, in models with endogenous separations à la Mortensen and Pissarides, as the cost to post a job vacancy goes to zero, either the economy converges to full employment or it features a 100% unemployment rate, i.e., labor-market shutdown. The full employment case is a standard property of the textbook DMP model in that frictional unemployment vanishes in the absence of recruiting costs. In the case of market shutdown, the 100% unemployment rate is due to rationing (in the sense that all matches would generate negative surplus). Obviously, this latter case is unrealistic. As such I conclude that models in the extended Mortensen-Pissarides class are unable to generate job rationing.

1.3.5 Aggregate Job-Finding Rate

In this section, I focus on the labor market in steady state, i.e., $z_t = z_s$ for $s \in \{1, \dots, \mathcal{S}\}$ and all t . This allows me to discuss the key properties of the aggregate job-finding rate in a transparent manner. The main goal of this section is to show

that permanent heterogeneity in workers' skill/productivity is the critical ingredient that enables the model to account for the volatility and asymmetry of actual job-finding rates. To this aim, in Section 1.3.5 below, I analyze a version of the model in which the labor market is integrated instead of (endogenously) segmented as presented in Section 1.3.1. This allows me to make a clear assessment of which margin, permanent heterogeneity versus segmentation, is the key for the qualitative and quantitative properties of aggregate job-finding rates. In fact, I argue that permanent heterogeneity, instead of segmentation, is the most relevant margin at work.

Segmented Labor Market

In this section, I study the steady-state properties of the aggregate job-finding rate in the segmented labor market as presented in Section 3.2. Let $\Phi(z_s)$ denote the aggregate job-finding rate which is a weighted average of job-finding probabilities specific to each submarket with weights equal to unemployment shares $\pi_s(x) \equiv u_s(x)/U_s$ where $U_s = \sum_x u_s(x)$ denotes aggregate unemployment:

$$\Phi(z_s) = \sum_x \pi_s(x) \phi(\theta_s(x)) = \frac{\overbrace{\sum_{x \leq \hat{x}(z_s)} u_s(x) \phi(\theta_s(x))}^{UE(x \leq \hat{x}(z_s)) = 0}}{\underbrace{\sum_{x \leq \hat{x}(z_s)} u_s(x)}_{U^R = \text{Non-employable workers}}} + \frac{\overbrace{\sum_{x > \hat{x}(z_s)} u_s(x) \phi(\theta_s(x))}^{UE(x > \hat{x}(z_s)) > 0}}{\underbrace{\sum_{x > \hat{x}(z_s)} u_s(x)}_{U^F = \text{Employable workers}}}.$$

(1.15)

At the numerator of equation (1.15), $UE(z_s) = UE(x \leq \hat{x}(z_s)) + UE(x > \hat{x}(z_s))$ are unemployment-to-employment (UE, hereafter) worker flows when the aggregate state of the economy is $s \in \{1, \dots, \mathcal{S}\}$. Note that $UE(x \leq \hat{x}(z_s)) = 0$ since low-skilled

workers below the rationing threshold $\hat{x}(z_s)$ in (1.11) are *non-employable workers* (in the sense that they generate negative surplus), i.e., $\phi(\theta_s(x)) = 0$ for all $x \leq \hat{x}(z_s)$. However, low-skilled, non-employable workers are in the unemployed pool, as such they count in the denominator of equation (1.15). This is the direct outcome of the selection mechanism that drives the endogenous separation decision. Low-skilled workers that are not viable for hiring are exactly the ones that have been laid off. This mechanism ties endogenous job destruction and job creation all together. Importantly, this is also the main point of departure from models of endogenous separations à la Mortensen and Pissarides (1994).

In Mortensen-Pissarides class of models, workers are identical, and heterogeneity across matches comes ex-post due to match-specific idiosyncratic shocks. Once matches are endogenously destroyed, workers flowing into the unemployment pool become viable for hiring. To make a closer analogy with the model of this paper, a Mortensen-Pissarides economy behaves *as if* laid-off workers become employable as soon as they enter the unemployment pool. As such the incentives to vacancy posting are larger and job-finding rates fall by less during downturns. In fact, the selection effect at work in the model of this paper is entirely due to the *permanent* nature of skills and it is also the only responsible for the large and deep falls of the aggregate job-finding rate during recessions.

Figure 1.5 shows the aggregate job-finding rate $\Phi(z_s)$ (see Panel A), UE worker flows $UE(z_s)$ (see Panel B), share of rationing unemployment, $u_s^R \equiv U_s^R / (U_s^R + U_s^F)$ (see Panel C), and share of frictional unemployment, $u_s^F \equiv U_s^F / (U_s^R + U_s^F)$ (see Panel D). The steady-state values are computed numerically with the same parameter values used for the quantitative analysis of Section 1.4. Panel A shows that aggregate job-finding rates are asymmetric. Same asymmetric behavior holds for UE worker flows in Panel B. These asymmetries derive from the fact that the number of non-employable workers in the unemployed pool (i.e., share of rationing unemployment

to total unemployment) sharply increases in response to a negative aggregate shock (see Panel C). The share of frictional unemployment instead decreases in recessions (see Panel D). Consistently with Michaillat (2012), the model suggests that the unemployment problem during recessions is caused by insufficient economic activity rather than matching frictions. However, differently from Michaillat's work, I argue that an increasing share of recessionary unemployment involves workers in the left tail of the skill/productivity distribution.

To further clarify the main point of this section, I rewrite the aggregate job-finding rate in (1.15) as follows:

$$\Phi(z_s) = \sum_{x > \hat{x}(z_s)} \pi_s(x) \phi(\theta_s(x)) \quad \text{with} \quad \pi_s(x) \equiv \frac{u_s(x)}{U_s}. \quad (1.16)$$

In principle, fluctuations in the aggregate job-finding rate $\Phi(z_s)$ come from movements in worker-specific shares of employable workers in the unemployed pool, $\pi_s(x)$, and worker-specific job-finding probabilities of employable workers, $\phi(\theta_s(x))$. To assess which margin is the most relevant, in the quantitative analysis of Section 1.4, I contrast two versions of the model: (i) endogenous job destruction (EJD) model which features endogenous separations and job rationing; and (ii) constant job destruction (CJD) model in which I calibrate the distribution of skills such that endogenous separations never occur in equilibrium. In this latter case, all workers in the unemployed pool are employable at any point of the business cycle. The quantitative results show that, different from the EJD model, the CJD model is unable to replicate the large and asymmetric fluctuations of the U.S. labor market. Thus, I conclude that fluctuations in the fraction of employable workers are of first-order importance for the quantitative success of the model.

As a final remark, note that during recessions the unemployment pool is characterized by a distribution of skills that is skewed to the left. This affects the aggregate

job-finding rate only through changes in the share of employable workers. Job-finding probabilities in each submarket are instead independent of the distribution of skills of the unemployed pool and only depend on the aggregate state of the economy. This latter result comes from the assumption that workers' types are observable to the employers at the time of vacancy posting such that the labor market is segmented.

Integrated Labor Market

In this section, I study the steady-state properties of the aggregate job-finding rate in the integrated labor market version of the model presented in Section 3.2. In this version of the model, the worker's type is known to the worker but it is unobservable by vacancy posting employers. Hence, an employer posting a vacancy to hire an unemployed worker internalize that it may come in contact with a worker of any type $x \in \{x_1, \dots, x_M\}$. The contact probability is taken parametrically by the employer and determined in equilibrium by the distribution of unemployment across worker types, $\pi_s = \{\pi_s(x_1), \dots, \pi_s(x_M)\}$ with $\pi_s(x) = u_s(x) / \sum_x u_s(x)$. Upon contact, the worker's type is revealed. At this stage, workers and employers decide whether to form a match and create a job or to continue their search process. The structure of the matching process implies that all unemployed workers face the same job-contact probability, $\phi(\theta_s(\pi_s))$, but different job-finding and separation rates. Notice that I explicitly denote the dependence of the tightness ratio $\theta_s(\pi_s)$ on the entire distribution of contact probabilities, π_s . I refer the reader to Appendix A.4 for further details on the model structure. Let $\Phi^{Int}(z_s)$ denote the aggregate job-finding rate in the integrated labor market:

$$\Phi^{Int}(z_s) = \frac{\phi(\theta_s(\pi_s)) \sum_{x > \hat{x}(z_s)} u_s(x)}{\sum_x u_s(x)} = \underbrace{\phi(\theta_s(\pi_s))}_{\text{Job-contact rate}} \times \underbrace{\sum_{x > \hat{x}(z_s)} \pi_s(x)}_{\text{Employable share of unemployed}}. \quad (1.17)$$

In principle, fluctuations in the aggregate job-finding rate $\Phi^{Int}(z_s)$ come from cyclical movements in job-contact probabilities, $\phi(\theta_s(\pi_s))$, and the share of employable workers in the unemployed pool, $\sum_{x > \hat{x}(z_s)} \pi_s(x)$. Notice that in the integrated labor market, worker-specific shares of the unemployed pool $\pi_s(x)$ affect the aggregate job-finding rate through two channels: (1) the distribution of worker-specific shares $\pi_s = \{\pi_s(x_1), \dots, \pi_s(x_M)\}$ directly affects market tightness $\theta_s(\pi_s)$. This channel is absent in the model with a segmented labor market. (2) The share of employable workers $\sum_{x > \hat{x}(z_s)} \pi_s(x)$ determines the fraction of workers in the unemployment pool that generate positive surplus once they come in contact with an employer. This channel is the only one at work in the model with a segmented labor market.

Figure 1.6 shows the aggregate job-finding, $\Phi^{Int}(z_s)$, and contact rate, $\phi(\theta_s(\pi_s))$ (see Panel A), UE worker flows (see Panel B), and the shares of rationing and frictional unemployment (see Panels C-D). Panel A shows that the aggregate job-finding rate (dotted line) displays a noticeable asymmetry. As such the job-finding rate in the economy with an integrated labor market shares the same asymmetric behavior of the economy with segmented labor markets (see Table 1.5, Panel A). Furthermore, as for the segmented labor-market case I discuss above, most of the asymmetry in the aggregate job-finding rate comes from the marked asymmetry in the share of non-employable workers in the unemployed pool (see Table 1.6, Panel C). Note that also the job-contact rate is asymmetric. However, its asymmetry is much less pronounced if compared to that of the job-finding rate. This result is important for

two reasons: (1) the *direct* effect from the composition of the unemployed pool to market tightness is not of first-order importance; and (2) the key mechanism driving asymmetry in the aggregate job-finding rate is the share of non-employable workers in the unemployed pool. Importantly, this is the key force at work in the model with segmented labor markets.

1.3.6 Aggregate Variables

Unemployment in submarket $x \in \{x_1, \dots, x_M\}$ follows the process:

$$u_{t+1}(x) = \begin{cases} 1 & \text{if } S_t(x) \leq 0 \\ u_t(x) + \delta e_t(x) - \phi(\theta_t(x))u_t(x) & \text{if } S_t(x) > 0. \end{cases} \quad (1.18)$$

Aggregate labor force is $\sum_x (e_t(x) + u_t(x)) = E_t + U_t = M$, where $E_t = \sum_x e_t(x)$ and $U_t = \sum_x u_t(x)$ denote respectively aggregate employment and unemployment. The aggregate employment and unemployment rates are respectively $e_t = E_t/M$ and $u_t = U_t/M$. The aggregate job-finding and job-separation rates are respectively,

$$\Phi_{t+1} = \frac{1}{U_t} \sum_x \phi(\theta_t(x))u_t(x) \quad \text{and} \quad \Delta_{t+1} = \frac{1}{E_t} \sum_x \delta_{t+1}(x)e_t(x),$$

where $\phi(\theta_t(x))$ and $\delta_t(x)$ are respectively job-finding and job-separation probabilities for workers of type x . Aggregate vacancies are $v_t = \sum_x v_t(x)$, where $v_t(x)$ is the number of vacancies posted in submarket x . Total output and aggregate labor productivity are respectively,

$$Y_t = \sum_x y_t(x) = z_t \sum_x x e_t(x) \quad \text{and} \quad p_t = \frac{Y_t}{E_t} = z_t \cdot \frac{\sum_x x e_t(x)}{\sum_x e_t(x)}.$$

Notice that aggregate labor productivity p_t consists of an exogenous component, z_t , and an endogenous component, $\sum_x x e_t(x) / \sum_x e_t(x)$, which is a skill-adjusted

measure of the employed pool.

1.3.7 Cross-Sectional Dispersion in Labor-Market Conditions and Matching Efficiency

The model features cross-sectional dispersion in labor-market conditions. Dispersion comes from the fact that different types of workers respond differently to aggregate shocks. This is a fundamental property of the model which endogenously arises since workers are permanently heterogeneous in skill/productivity. I next show that if an econometrician wrongly assumes the existence of an aggregate matching function then she would infer cyclical movements in matching efficiency. This would be misleading since in the model the technology parameter governing the efficiency of the market-specific matching function, μ , is constant. Recall that for each submarket the number of hires is $m_t(x) = \mu v_t(x)^\alpha u_t(x)^{1-\alpha}$. In the aggregate instead, total hires are $M_t = \sum_x m_t(x)$,

$$M_t = \mu \sum_x v_t(x)^\alpha u_t(x)^{1-\alpha}, \quad (1.19)$$

which is the true data generating process (DGP). Now, if an econometrician assumes the existence of an aggregate matching function then she would expect the following relationships to hold,

$$\tilde{M}_t = \tilde{A}_t \left(\sum_x v_t(x) \right)^\alpha \left(\sum_x u_t(x) \right)^{1-\alpha} = \tilde{A}_t V_t^\alpha U_t^{1-\alpha}, \quad (1.20)$$

where \tilde{M}_t are the number of hires implied by an aggregate matching function, $V_t = \sum_x v_t(x)$ and $U_t = \sum_x u_t(x)$ are respectively aggregate vacancies and unemployment, and \tilde{A}_t is measured matching efficiency. Suppose the econometrician is endowed with a dataset of artificial data generated by the model, $\{M_t, V_t, U_t\}_{t=0}^T$, and the value of the parameter α . I ask now the fictional econometrician to estimate matching

efficiency \tilde{A}_t without providing her the true DGP in equation (1.19). She would calculate efficiency in a residual way as

$$\ln \tilde{A}_t = \ln M_t - \alpha \ln V_t - (1 - \alpha) \ln U_t. \quad (1.21)$$

Concavity of the matching function and Jensen's inequality imply that $M_t \leq \tilde{M}_t$ such that $\ln \tilde{A}_t \leq 0$ for all $t \geq 0$. Therefore, cross-sectional dispersion in tightness ratios induces a negative level effect on measured matching efficiency as compared to the benchmark economy with homogenous workers. Table 1.7 shows that, in addition to a level effect, measured matching efficiency would also display cyclical properties.

However, these inferred variations are the artifact of neglecting heterogeneity. If the econometrician is endowed with the DGP in equation (1.19) and market-specific data on job vacancies and unemployment, $\{v_t(x), u_t(x)\}_{t=0}^T$, then she would estimate $\ln \tilde{A}_t = \mu$ for all $t \geq 0$.

The model suggests that recessions would look like periods of extremely low matching efficiency. The model generates an high correlation between measured matching efficiency and the aggregate shock, i.e., $\text{corr}(\ln \tilde{A}_t, \ln z_t) = 0.792$. Elsby et al. (2010) and Barnichon and Figura (2011) provide empirical support for this prediction. Using CPS micro data for 1976-2010, Barnichon and Figura (2011) show that the composition of the pool of unemployed accounts for most of the cyclical variation in matching efficiency up to 2006, and forty-five percent of the decline in matching efficiency for the 2007-2010 period. Note that in the model, dispersion in labor-market conditions across workers depends only on their different responses to aggregate shocks. Hence, there is no room for "mismatch." By mismatch I mean the sectoral misalignment between vacant jobs and unemployed workers, i.e., unemployed seeking employment in sectors (occupations, industries, locations) different from those where the available jobs are. Sahin et al. (2012) and Herz and van Rens

(2012) argue that fluctuations in unemployment due to mismatch are small compared to the overall unemployment rate. As a final remark, the discussion of this section also suggests that interpreting cyclical movements in matching efficiency as “*structural shocks*” to the matching process can lead to erroneous explanations of unemployment dynamics during recessions.

1.3.8 Textbook DMP Model

In this section, I discuss the properties of the textbook DMP model which is a special case of the model with worker heterogeneity.

Remark 1 (Textbook DMP model). *When $x_j = 1$ for all $j \in \{1, \dots, M\}$, all workers are identical. In this case, the model nests a standard DMP model with a constant exogenous rate of job destruction.*

The analysis focuses on the stochastic equilibrium of the economy. This allows me to study the equilibrium analytically.

Definition 7 (Stochastic equilibrium). *A stochastic equilibrium is any equilibrium in which the shock $\{z_t\}$ repeats itself, i.e., $z_{t+1} = z_t = z_s$, for $s \in \{1, \dots, \mathcal{S}\}$.*

At the stochastic equilibrium, the employment rate is

$$e_s = \frac{\phi(\theta_s)}{\delta + \phi(\theta_s)}.$$

Employment rate, e_s , job-finding rate, $\phi(\theta_s)$, vacancies, $v_s = \theta_s u_s$, and log output, $\ln y_s$, are strictly increasing and concave functions of the tightness ratio, θ_s .

1.3.9 Three Propositions

I next derive analytical results for the stochastic equilibrium of the standard DMP model.

Proposition 8 (Asymmetry bounds). *Any stochastic equilibrium consistent with a constant exogenous rate of job destruction features asymmetry bounds on the unconditional distributions of the endogenous variables: for market tightness, (i) $\Delta\theta_{MMD} \equiv \mathbb{E}[\theta] - \theta_{sm} > 0$; and for any strictly increasing and concave function $\xi(\theta)$ (i.e., employment rate, job-finding rate, vacancies, and log of output), (ii) $\Delta\xi_{MMD} \equiv \mathbb{E}[\xi(\theta)] - \xi(\theta_{sm}) < \xi(\mathbb{E}[\theta]) - \xi(\theta_{sm}) < \xi'(\theta_{sm})\Delta\theta_{MMD}$.*

Proof. See Appendix A.3. ■

Proposition 8 formally states that the asymmetry properties of the endogenous variables are bounded by those of market tightness. Despite the theoretical convexity proved by Lemma 22 in Appendix A.3, the relationship between market tightness and aggregate shock turns out to be approximatively linear for all plausible calibrations of the model. This latter observation motivates the next corollary.

Corollary 9 (Mean-median asymmetry in levels). *Consider the stochastic equilibrium for market tightness. If the support of the distribution of θ is symmetric around the median value then any equilibrium consistent with a constant exogenous rate of job destruction features negative skewness in the unconditional distribution of employment rate, job-finding rate, vacancies and log of output.*

Proof. See Appendix A.3. ■

The results in Corollary 9 derive from a key property of the search-and-matching framework, i.e., matching displays decreasing returns to unemployment and vacancy posting. When the labor market is tight— θ is high—increasingly more vacancies are needed to generate a given reduction in unemployment. This is the essence of the congestion externality which drives fluctuations in DMP models.

Proposition 10 (Volatility and mean-median asymmetry in levels). *Consider a mean-median-preserving spread Δ in the distribution of market tightness with*

stochastic equilibrium, $\theta_s = \theta_{s_m} + \frac{(s-s_m)}{(\mathcal{S}-s_m)} \cdot \Delta$, for $s \in \{1, \dots, \mathcal{S}\}$ and $\Delta > 0$. In any stochastic equilibrium consistent with a constant exogenous rate of job destruction, as Δ increases, the unconditional distribution of employment rate, job-finding rate, vacancies and log of output become more negatively skewed.

Proof. See Appendix A.3. ■

Notice that by construction Δ does not affect the symmetry of the distribution of the tightness ratio, but it controls its range of variation. Hence, Proposition 10 characterizes a tight link between volatility and asymmetry in levels. Notice also that the proposition is built on the properties of the key endogenous variable of the model, i.e., market tightness, implying that any mechanism, be it exogenous or endogenous, raising the volatility of the tightness ratio also affects the asymmetry properties of the model. Specifically, more volatility induces more asymmetry in levels. Figure 1.8 shows a numerical example of the analytical results stated in Proposition 10.

In Figure 1.8, the inner distributions (red lines) are associated to the least volatile tightness ratio. The outer distributions (purple lines) are associated instead to the most volatile tightness ratio. Two features of the theoretical distributions are worth further explanation. First, in Panel A, notice that despite an evident increase in volatility, the distributions of tightness ratios preserve a substantial symmetry. This happens because the tightness ratio is approximately linear in the shock, see Panel A in Figure 1.9. Second, as the dispersion in the distribution of the tightness ratio increases, the distributions of the endogenous variables become increasingly left-skewed. This happens because employment rate, job-finding rate, vacancies and output are concave functions of the tightness ratio, see Figure 1.9. Notice that asymmetries arise with perfectly symmetric shocks, see Panel F in Figure 1.8.

I next characterize the properties of the endogenous variables in first-differences.

Proposition 11 (Mean-median asymmetry in first-differences). *Consider the stochastic equilibrium of a generic variable of the model \tilde{x}_s , for $s \in \{1, \dots, \mathcal{S}\}$. At any equilibrium of the model consistent with a constant exogenous rate of job destruction:*

- (i) if the support of the distribution of \tilde{x}_s is symmetric around the median, \tilde{x}_{sm} , then the unconditional distribution of $\Delta\tilde{x}_{s,s'}$ is symmetric;*
- (ii) if the support of the distribution of \tilde{x}_s has a longer left tail, then the unconditional distribution of $\Delta\tilde{x}_{s,s'}$ is right-skewed;*
- and (iii) if the support of the distribution of \tilde{x}_s has a longer right tail, then the unconditional distribution of $\Delta\tilde{x}_{s,s'}$ is left-skewed.*

Proof. See Appendix A.3. ■

Tables A.6 and A.7 show numerical results which further strengthen the theoretical predictions of this section. The numerical analysis is based on two alternative calibrations of the worker's outside option. In the first calibration, which I refer to as Hall and Milgrom (2008)-type calibration, the worker's outside option amounts to 73% of steady-state wage. In the second calibration, which I refer to as Hagedorn and Manovskii (2008)-type calibration, I set the worker's outside option to 95% of steady-state wage. It is well-known that a textbook DMP model with a Hall-Milgrom-type calibration fails to generate realistic amplification in response to exogenous impulses. On the other hand, Hagedorn-Manovskii-type calibrations greatly help the standard model to generate realistic unemployment fluctuations. The numerical results in Appendix B.1 suggest that a textbook DMP model with a constant exogenous rate of job destruction *cannot* account for the asymmetry properties of the data irrespective of its ability to generate realistic volatility.

1.4 Quantitative Analysis

In this section, I assess the quantitative properties of the model.

1.4.1 Calibration

The model is calibrated at the monthly frequency, as summarized in Table 1.3. The discount rate is set to $\beta = 0.9959$ to accord with an annual risk-free interest rate of 5%. I next discuss calibration of the labor-market parameters: α and μ for the matching function, worker's Nash-bargaining weight, η , vacancy costs, $k(x)$, exogenous and constant job destruction rate, δ , and worker's outside option, λ . Relative to the standard DMP model, the model adds a number of new parameters: x_j , for $j = 1, \dots, M$.

I assume a Cobb-Douglas matching function, such that the job-finding rate, $\phi(\theta)$, and job-filling rate, $\rho(\theta)$, are in the following relation, $\phi(\theta) = \theta\rho(\theta) = \mu\theta^\alpha$. A large literature that directly estimates the aggregate matching function, provides a range of estimates for the parameter α . Petrongolo and Pissarides (2001) establish a plausible range of 0.3-0.5. Brügemann (2008) obtains a refined range of 0.37-0.46. I specify $\alpha = 0.4$ at the mid point of these ranges. For comparability with previous work, I specify the parameter of the Nash-bargaining problem as $\eta = 1 - \alpha$ such that the Hosios (1990)'s condition is met and the decentralized equilibrium is efficient. I refer to Shimer (2005) for further details.

I set the matching function scale to $\mu = 0.33$ such that the model-implied median unemployment rate equals 5.6%, which matches the median unemployment rate in the U.S. data for 1948:Q1-2011:Q3. The seasonally-adjusted monthly U.S. unemployment rate series is constructed by the BLS from the CPS. I set the constant exogenous rate of job destruction to $\delta = 0.0189$, which is the average quit rate in the U.S. data for 2001:M1-2011:M9. The seasonally-adjusted monthly series for total quits to employment in the nonfarm business sector is constructed by the BLS from JOLTS.

I choose to target a v-u ratio of 1 in each submarket, which requires setting

vacancy costs to $k(x) \in [0.0005, \dots, 0.3359]$. This calibration strategy implies that the cost to post a vacancy is (approximately) linearly increasing in skills, i.e., $k(x) = kx$. This is broadly consistent with the empirical evidence reported by Hamermesh and Pfann (1996).

I assume that workers' productivity distribution is uniform, satisfying $x_1 = \underline{x}$, $x_M = \bar{x}$, and $x_j - x_{j-1} = (\bar{x} - \underline{x})/M$. Thus, $j = M$ is the most productive, and $j = 1$ is the least productive submarket. I choose $M = 200$ and normalize $\bar{x} = 1$. For the lower bound, I set $\underline{x} = 0.44$, such that the submarket at the 90th percentile of the distribution is twice as productive as the submarket at the 10th percentile, $\ln(x_{180}/x_{20}) = 0.651$. Syverson (2011) finds productivity differences of this order of magnitude within four-digit SIC industries in the U.S. manufacturing sector.

I set the flow value of unemployment to $\lambda = 0.44$ such that all submarkets feature positive match surpluses in steady-state ($z_{sm} = 1$ in my normalization), as such steady-state unemployment is all frictional. This calibration implies an average replacement ratio of 62%, which is a value smaller than those used in Hall and Milgrom (2008), Mortensen and Nagypal (2007) and Pissarides (2000), and much smaller than Hagedorn and Manovskii (2008)'s calibration. In this regard, this paper's calibration is a conservative one.

Calibration of the worker's outside option, λ , deserves further explanation. The assumption that λ is the same across workers of different skills implies that replacement ratios, i.e., λ as percent of the wage, greatly differ across workers' types. This happens because low-skilled workers earn lower wages. Hence, while on average λ amounts to 62% of aggregate wage compensation, there is a spectrum of replacement ratios. In Figure 1.10, Panel A shows the CDF of replacement ratios. Notice that approximately 5% of the aggregate labor force features a Hagedorn-Manovskii-type calibration, and approximately 70% of the labor force have replacement ratios smaller than those implied by a Hall-Milgrom-type calibration. Panel B shows

that replacement ratios are decreasing in workers' productivity. Specifically, the 5% of Hagedorn-Manovskii-type of workers are the least productive in the labor force. These low-skilled workers are the ones benefiting relatively less from being employed and always at risk of layoff. They are necessarily marginal workers in that they are almost indifferent to work in the labor market and enjoy the value of non-market activities.

Finally, I calibrate the exogenous stochastic process for labor productivity, which is the only driving force of fluctuations. I estimate an AR(1) process for the HP-filtered seasonally-adjusted quarterly real output per worker in the nonfarm business sector constructed by the BLS from the LPC release, for 1948:Q1-2011:Q3: $\ln(z_{t+1}) = \rho_z \ln(z_t) + \sigma_\epsilon \epsilon_{t+1}$ with $\epsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. The HP-filter smoothing parameter is 10^5 . With quarterly data, we obtain an autocorrelation of $\hat{\rho}_z = 0.8963$ and a residual standard deviation of $\hat{\sigma}_\epsilon = 0.0091$, which yields $\hat{\rho}_z = 0.8963^{1/3} = 0.9642$ and $\hat{\sigma}_\epsilon = 0.0055$ at monthly frequency. Following Tauchen (1986), I approximate the continuous-valued AR(1) process for $\ln(z_t)$ through a \mathcal{S} -state Markov chain, having a discrete state space $\{z_1, \dots, z_{\mathcal{S}}\}$ and transition probabilities $\pi_{s,s'} = \Pr\{z_{t+1} = z_{s'} | z_t = z_s\}$. I set the number of grid points for the state space to $\mathcal{S} = 9$.

1.4.2 Simulated Moments

In this section, I assess the quantitative performance of the model by comparing important simulated moments to their empirical counterparts in the U.S. data. I contrast two versions of the model which differ for the calibration of the lower bound of the workers' productivity distribution: (1) endogenous job destruction (EJD) model. In this case, $\underline{x} = 0.438$, and the model features an endogenous rate of job destruction as in (1.10); (2) constant job destruction (CJD) model. In this case, $\underline{x} = 0.455$, and endogenous job separations never occur in equilibrium. Notice that the CJD model can be seen as a collection of standard DMP models with

constant exogenous rates of job destruction and cross-sectional dispersion of total match surpluses, i.e., $S_s(\underline{x}) < S_s(x_2) < \dots < S_s(\bar{x})$. Comparing these two versions of the model highlights the key ingredients for the quantitative success of the model.

Volatility and Comovement in U.S. Data and Model

Table 1.4 shows standard deviations (relative to labor productivity) and correlations for labor-market variables in the model and U.S. data. Panel A forcefully shows that the EJD model generates fluctuations in the unemployment rate, vacancies and job-finding rates that are comparable to those in the U.S. data. The EJD model outperforms the CJD model in any dimension. In this regard, the results for the CJD model confirm the previous negative findings on the inability of the standard DMP model to amplify exogenous impulses. The findings suggest that key for the amplification properties of the model is the selection mechanism which drives endogenous job destruction and job creation.

In Table 1.4, Panel B shows comovements between labor-market variables. Except for the correlation between vacancies and unemployment (Beveridge curve), the EJD model outperforms the CJD model. However, it still produces correlations of unemployment, job-finding rates and vacancies with labor productivity that are too high (in absolute value) compared to the data. The success of the CJD model in replicating the empirical Beveridge curve is not surprising given its resemblance to a textbook DMP model. Shimer (2005) shows that the Beveridge curve is the only quantitative success of a textbook DMP model.

To understand the comovement between vacancies and unemployment, Figure 1.11 shows lead-lag correlations, where the current period unemployment rate is associated with future and lagged values of vacancies up to four quarters. First observe that a large value of contemporaneous correlation between unemployment and vacancies observed in the U.S. data, -0.858 , is reasonably close to the value generated

by the model of -0.561 . The model preserves a downward-sloping Beveridge curve. A well-known criticism to models with endogenous job separations à la Mortensen and Pissarides (1994), is their counterfactual implications for the Beveridge curve. These models produce a strong positive correlation of unemployment with vacancies, whereas in the U.S. data this correlation is strongly negative at business cycle frequencies. With respect to the lead-lag relationship, the data suggest some tendency for vacancies to lead unemployment.¹⁵ Qualitatively, this pattern is captured well in the model. This reflects the mechanics of the model, wherein the search friction produces some lagged response in unemployment after the response in vacancy posting. In Mortensen and Pissarides’s type of models, in contrast, the feedback from the movement of the separation rate into vacancy posting erases this feature, generating the tendency that unemployment leads vacancies.

Asymmetry in U.S. Data and Model

Table 1.5 contrasts the model with the asymmetry properties of the data. The EJD model generates asymmetries in employment, output and job-finding rates comparable to those observed in the data. Not surprisingly, given the results in Section 1.3.8, the CJD model completely fails to do so. Remarkably, the EJD model is also able to replicate the disconnect between the asymmetry properties of employment and output, i.e., it generates deep and steep cycles in employment, and deep cycles in output with no steepness.

The model produces counterfactual implications for the asymmetry properties of vacancies, i.e., positive instead of negative skeweness in level and growth rates. These counterfactual predictions are due to the “echo effect” that characterizes the dynamics of vacancies in models with endogenous job separations. After a adverse

¹⁵ Correlations between lagged values of vacancies and current unemployment tend to be larger (in absolute value) than those between future values of vacancies and current unemployment.

aggregate shock that induces endogenous destruction of jobs, the pool of unemployed raises, the probability to fill a vacancy increases such that for the employer the expected recruiting cost decreases. This gives to employers strong incentives to post vacancies right at the end of a downturn since it is relatively cheaper to do so. Because in the model vacancies are a jump variable, they react “too much and too fast” to positive changes in the aggregate state coming from a recession causing positive skewness in levels and growth rates.

What drives asymmetric employment dynamics?

In this section, I assess the relative contribution of job-finding and job-separation rates to the skewness asymmetry of the employment rate series generated by the model. To this aim, I construct two counterfactual series for the employment rate in the EJD model. Recall that aggregate employment and employment rate are respectively $E_t = \sum_x e_t(x)$ and $e_t = E_t/M$. The first counterfactual is an employment rate series that only allows for variation in job-finding rates, $e_{t+1}^{\text{jfr}} = E_{t+1}^{\text{jfr}}/M$, where

$$E_{t+1}^{\text{jfr}} = E_t^{\text{jfr}} + \sum_x \phi(\theta_t(x)) u_t^{\text{jfr}}(x) - \sum_x \delta_{s_m}(x) e_t^{\text{jfr}}(x),$$

and $\delta_{s_m}(x) = \delta$ are steady-state job-separation rates which are calibrated to be the same across submarkets. The second counterfactual series only allows for variation in job-separation rates, $e_{t+1}^{\text{jsr}} = E_{t+1}^{\text{jsr}}/M$, where

$$E_{t+1}^{\text{jsr}} = E_t^{\text{jsr}} + \sum_x \phi(\theta_{s_m}(x)) u_t^{\text{jsr}}(x) - \sum_x \delta_{t+1}(x) e_t^{\text{jsr}}(x),$$

and $\phi(\theta_{s_m}(x)) = \mu$ are steady-state job-finding rates which are calibrated to be the same across submarkets. Which counterfactual series better accounts for the asymmetry properties of the employment rate? Recall I asked the same question in Section 1.2 for the actual U.S. employment rate. Answering this question allows us

to disentangle the contribution of job-finding and separation rates to the skewness in levels and growth rates. In Table 1.6, Panel A shows that both margins of the labor market, i.e., job-finding and job-separation rates, are jointly responsible for the skewness in levels of the employment rate. This prediction holds in U.S. data (see Fact 3 in Section 1.2).

Panel B shows that in the model, job-separation rates are the only responsible for negative skewness in growth rates. Moreover, when the dynamics are only driven by job-finding rates, the model generates positive instead of negative skewness in growth rates. Importantly, these model predictions hold in the data (see Fact 4 in Section 1.2). To summarize, this section strengthens the results in Section 1.4.2 in that it shows the model is able to replicate not only the skewness asymmetry in employment but also the asymmetry properties implied by job-finding and separation rates.

What drives asymmetric output dynamics?

To understand what drives the disconnect between the asymmetry properties of employment and output, Table 1.7 shows statistics for the skewness in levels and growth rates for each output component simulated from the EJD model. In the model, aggregate output is

$$Y_t = \sum_x y_t(x) = z_t \cdot \sum_x x e_t(x),$$

where z_t is the exogenous aggregate shock, and $\sum_x x e_t(x)$ is a quality-adjusted measure of aggregate employment, which is a weighted average of worker-specific employment rates with weights equal to workers' types $x \in \{x_1 < \dots < x_M\}$. In Table 1.7, the column labeled Y reproduces the skewness statistics for aggregate output in Table 1.5. As discussed in Section 1.4.2, in the model as in the data, aggregate output displays negative skewness in levels with nearly no skewness in growth rates.

Recall that the aggregate state z_t is by assumption a symmetric Markov process, as such simulated shocks display (approximately) zero skewness both in levels and growth rates. Note that quality-adjusted employment, $\sum_x xe(x)$, displays negative skewness in levels and growth rates. Hence, it inherits the asymmetry properties of aggregate employment (see Table 1.5).

The results in Table 1.7 suggest the reason why aggregate output behaves more symmetrically than employment is that aggregate shocks are symmetric and hit the production function directly.

1.4.3 *Impulse Response Functions*

In this section, I discuss key properties of the model by the means of impulse response functions (IRFs). I derive the dynamic response to a productivity shock by comparing the expected paths of two economies. The first economy starts with the level of the endogenous variable associated with the median state but is in the state below (above) the median state. The second economy starts with the same level of the endogenous variable and is in the median state. The difference between the two paths is the response over time to a negative (positive) impulse, which is the transition from the median to lower (higher) states that occurred at time zero. I report the average response across 5,000 replications. Critically, the size of the positive and negative shock is the same. Figure 1.12 shows the responses of labor-market variables to a negative (solid line) and positive (dashed line) impulse. I flip the sign of the response to a positive impulse (dashed line) such that both responses lie in the same quadrant. Panel D shows the responses of the exogenous aggregate shock, which is by assumption a symmetric stochastic process.

In Figure 1.12, Panel A shows that the employment rate's response to shocks is strongly asymmetric, with the response to a negative impulse being stronger than the response to a positive impulse. What is driving the asymmetric dynamics of

the employment rate? Suppose at time $t = 0$ the economy rests at the median state ($z_{sm} = 1$ in my normalization), and a negative shock realizes. The immediate response of the model economy is a burst of job destruction which leads a spike in the aggregate separation rate, see Panel B. The endogenous job destruction margin is the one contributing more to the speed at which the employment rate reaches the trough of the response. At this stage, the aggregate job-finding rate plays a minor role in the dynamics. After the trough of the response is reached, the dynamics are governed by a low aggregate job-finding rate which determines the slow recovery towards the initial state. At this stage, job rationing is at work. The employment rate's response to a positive shock is driven only by a high aggregate job-finding rate. In this case, the forces driving the dynamics are the same as those of a standard DMP model with a constant job-separation rate. Panel C shows that the IRFs of the aggregate job-finding rate are also strongly asymmetric. The aggregate job-finding rate falls in response to a negative impulse much farther than it raises in response to a positive impulse. This asymmetric response comes from the direct effect that endogenous job destruction exerts on the aggregate job-finding rate. After a negative shock, job-finding probabilities of all workers fall, however, they drop disproportionately more for low-skilled workers previously fired which are over-represented in the group flowing into unemployment. Notice that the asymmetric behavior of the labor market comes entirely from the mechanics of the model. Panel D shows that the IRFs of the exogenous aggregate shock are fully symmetric. In the model, aggregate output is $Y_t = \sum_x y_t(x) = z_t \cdot \sum_x x e_t(x)$. Labor productivity, Y_t/E_t , consists of an exogenous and endogenous component,

$$\frac{Y_t}{E_t} = \underbrace{z_t}_{\text{Aggregate shock}} \cdot \underbrace{\frac{\sum_x x e_t(x)}{\sum_x e_t(x)}}_{\text{"Composition effect"}}. \quad (1.22)$$

The first term on the right hand side of equation (1.22) is the exogenous aggregate shock, z_t . The second term on the right hand side—“composition effect”—is a skill-adjusted measure of employment, which is an endogenous variable. Figure 1.13 shows the impulse responses for output, labor productivity, and the composition effect.

In Figure 1.13, Panel A shows that output contracts deeper after a negative impulse than it expands after a positive shock, i.e., the trough (in absolute value) exceeds the peak. However, besides the peak-trough asymmetry, the impulse responses are more symmetric if compared to those of the employment rate (see Panel A in Figure 1.12). Panel B shows the response of labor productivity. The sharp fall in employment after a negative impulse is muted by the endogenous response of labor productivity, which tends to fall by less because of the composition effect. Panel C shows that the composition effect’s responses to shocks are strongly asymmetric. It reacts much stronger to a negative than to a positive impulse. Specifically, it raises more in response to a negative impulse than it falls after a positive impulse. After a negative impulse, low-skilled workers are laid off, a disproportionate part of the employed pool consists of high-skilled workers. This selection mechanism causes skill-adjusted employment to raise sharply. After a positive impulse instead, no endogenous job destruction occurs, a disproportionate part of the employed pool consists of low-skilled workers. In this case, the composition effect causes skill-adjusted employment to smoothly decrease.

1.5 Labor-Market Fluctuations at the Micro Level

The model highlights the crucial role of low-skilled workers in shaping cyclical movements in aggregate unemployment. As a by-product, the model provides stark predictions for the time series of skill-specific unemployment rates. Precisely, low-skilled workers experience (i) higher average unemployment rates and (ii) they account for most of the variation in aggregate unemployment. In this section, I use CPS mi-

cro data for the period 1976:M1-2013:M2 and test this prediction. Specifically, the goal is to investigate whether there are relatively unskilled groups of workers that contribute disproportionately more to the large fluctuations of the U.S. labor market.

Clearly, the identification of worker's abilities in the data is challenging. However, a large empirical literature in the tradition of Mincer (1974)'s work, has identified *experience* and *education* as important drivers of wage differentials. Hence, to the extent to which actual wages reflect workers' marginal product, one can arguably think of age and education as good proxies for worker-specific productivity on the job. Following the lead of this literature, I consider unemployment rates by age and education groups and investigate whether their time series properties accord with the predictions of the model. To this aim, I consider 6 age groups (16-24, 25-34, 35-44, 45-54, 55-64, 65 and over) and 3 education levels (High school dropouts, HSD, High school graduates joint with some college, HSG/SC, and College graduates and post-college degree holders, CGPC). As such there are 18 age/education groups. The first row of Table 1.8 reports average unemployment rates for different age groups.

Unemployment rates decrease monotonically with age. This fact is well-known, see Gervais et al. (2013). Specifically, the average unemployment rate for the 16-24 years old is 12.9% and sharply falls to 3.92% for the 55-64 years old. Note also that the age differences are large. The second row of Table 1.8 displays the average unemployment rate for each age group relative to that of the 45-54 years old. Over the period 1976:M1-2013:M2, the average unemployment rate for the 16-24 years old was 3.16 times that of the 45-54 years old.

Table 1.9 displays average unemployment rates for different education groups. Clearly, unemployment rates sharply decrease with education. The average unemployment rate for high-school dropouts is 12.72% and falls to 2.79% for workers with the highest educational attainments. Note that the average unemployment rate for HSD is 2 times that of the HSG/SC and as high as 4.56 times that of workers with

a college or post-college degree.

Table 1.10 reports average unemployment rates for different age/education groups. The results show that unemployment rates are decreasing in age and education even within groups. Precisely, (i) average unemployment rates decrease monotonically with age after sorting on education levels and (ii) average unemployment rates decrease monotonically with education after sorting on age differences. These facts are important since confirm a tight monotonic link between unemployment rates and skill levels. Note also that age/education differences are large. High-school dropouts in the 16-24 years old group experience an average unemployment rate of 21% which is 9 times that of the CGPC workers in the 45-54 years hold group.

Overall, the empirical findings suggest that the U.S. labor market features large differences in average unemployment rates once I sort workers by age and education. These facts are broadly consistent with the predictions of the model. Table 1.11 displays average unemployment rates by skill/productivity group for artificial data generated by the model. Recall that the index $x \in \{x_1, \dots, x_M\}$ denotes the skill type of the worker which determines her productivity on the job. The model generates large dispersion in average unemployment rates across workers. Specifically, lowest-skilled workers experience an average unemployment rate 8.5 times larger than that of high-skilled workers in the right tail of the productivity distribution, i.e., $(x_7 - x_{200})$ group. Importantly, this dispersion in unemployment rates is the endogenous outcome of the model. I calibrated the unemployment rates to be the same across workers in the median state ($z_{s_m} = 1$ in my normalization). As such these large differences in average unemployment rates come exclusively from the different cyclical behavior of workers with different skills.

I next provide evidence that the young and least-educated workers not only experience higher average unemployment rates but they also contribute disproportionately more to the time series variation of the U.S. unemployment rate.

To this aim, let us decompose the actual U.S. unemployment rate as the sum of unemployed workers sorted in different groups indexed by x divided by the aggregate labor force, $lf_t \equiv E_t + U_t$, such that

$$u_t = \frac{U_t}{E_t + U_t} = \frac{u_t(x_1)}{E_t + U_t} + \frac{u_t(x_2)}{E_t + U_t} + \dots + \frac{u_t(x_M)}{E_t + U_t} \quad (1.23)$$

where $U_t = \sum_x u_t(x)$ and $E_t = \sum_x e_t(x)$ are respectively the total number of unemployed and employed. The index $x \in \{x_1, \dots, x_M\}$ denotes an observable worker-specific characteristics, e.g., age and/or educational attainments. Let $\hat{u}_t(x) \equiv u_t(x)/lf_t$ denote the share of unemployed workers of characteristic x in the labor force. Standard calculations yield

$$Var(u_t) = \sum_x Var(\hat{u}_t(x)) + \sum_{x \neq x'} Cov(\hat{u}_t(x), \hat{u}_t(x')). \quad (1.24)$$

Tables 1.12-1.13 report percentage shares of U.S. unemployment rate variance attributed to a specific group of workers. Specifically, I group workers by age (see Table 1.12), education (see Table 1.13), and age/education (see Table A.8 in Appendix B.1).

Table 1.12 shows that 2 age groups, i.e., 16-24 and 25-34, account for the bulk of the variation in the actual U.S. unemployment rate. A similar picture emerges from Table 1.13 in which HSD, HSG and SC are responsible for most of the time series variation of U.S. unemployment. Table A.8 in Appendix B.1 further confirms that in the U.S. economy, unemployment volatility is clustered between the young and the least-educated workers.

Importantly, these findings are consistent with the predictions of the model. In Table 1.14, I report volatility shares for artificial data generated by the model. In the model, as in the data, few skill/productivity groups account for most of the variation

of the unemployment rate. These are workers in the left tail of the skill/productivity distribution.

Note that changes in unemployment shares $\hat{u}_t(x) \equiv u_t(x)/lf_t$ are due to changes in either skill-specific unemployment rates, or skill-specific shares of the aggregate labor force:

$$\hat{u}_t(x) = \underbrace{\frac{u_t(x)}{e_t(x) + u_t(x)}}_{\text{Skill-specific unemployment rate}} \times \underbrace{\frac{e_t(x) + u_t(x)}{E_t + U_t}}_{\text{Skill-specific shares of the labor force}}. \quad (1.25)$$

I refer to the first margin as the *unemployment margin*, and to the latter as the *participation margin*. If the participation margin was the main driver of fluctuations in unemployment shares of young and/or least educated workers then modeling skill-specific differences in participation decisions would be of first-order importance. In this case, my modelling choice of constant labor force shares would be unappealing. If not, it would suggest that to a first-order, the factor generating age/education group differences are variations in skill-specific unemployment rates. This would bring support to the main mechanism at work in the model. The variance of unemployment shares is decomposed as:

$$Var(\hat{u}_t) = Var(ur_t(x)) + Var(lfs_t(x)) + 2Cov(ur_t(x), lfs_t(x)), \quad (1.26)$$

where $ur_t(x) = u_t(x)/(e_t(x) + u_t(x))$ and $lfs_t(x) = (e_t(x) + u_t(x))/(E_t + U_t)$. Tables 1.15-1.16 show that almost all variation at business cycle frequencies in unemployment shares comes from fluctuations in skill-specific unemployment rates. Table A.9 in Appendix B.1 shows the unemployment shares' variance decomposition by age/education group.

To summarize, the facts documented in this section state that: (i) most of the time series variation in the U.S. unemployment rate can be attributed to the young and least-educated workers. Since age and education are natural proxies for skills, the empirical findings suggest that understanding the cyclical behavior of low-skilled workers is critical to explain the large fluctuations of the U.S. labor market; (ii) at business cycle frequencies, most of the volatility in unemployment shares comes from changes in skill-specific unemployment rates and not from variation in skill-specific shares of the aggregate labor force. These facts are reminiscent of the well-known observation that volatility in labor market conditions greatly differs for workers of different age and education levels (see Clark and Summers (1981), Gomme et al. (2005), and Jaimovich and Siu (2009) among others). Importantly, this latter result also suggests that modelling skill-specific differences in participation decisions is not of first-order importance. Overall, the empirical analysis provides strong support for the main prediction of the model: the left tail of the skill/productivity distribution is key to understand unemployment dynamics.

1.6 Policy Analysis

In this section, I show that in addition to matching key business cycle moments, the model also generates a wide range of implications for the design of macroeconomic policies. For instance, a general prediction of the analysis is that the effectiveness of macroeconomic policies varies over the business cycle. For instance, policies apt to decrease matching frictions in the labor market are less effective during recessions than during booms: programmes designed to give job search assistance to the unemployed (e.g., think of an exogenous increase in matching efficiency) and/or policies apt to decrease recruiting costs are far less effective in reducing unemployment during recessions than expansions. This happens because during recessions, jobs are rationed: matches with low-skilled workers would generate negative surpluses even

if the cost to post a job vacancy was zero (i.e., no recruiting costs). Note that the ineffectiveness of these type of policies in a search-and-matching framework with rationing has been first discussed by Michaillat (2012). However, in Michaillat’s model workers are identical and rationing arises in recessions because of decreasing returns to labor and wage stickiness. In this paper instead, rationing is a by-product of worker permanent heterogeneity. Hence, differently from Michaillat’s work, I argue that low-skilled workers are the ones bearing the cost of this policy failure. The main insight of the analysis is that any policy that leaves un-changed the surplus after the match has occurred, it is then ineffective in reducing unemployment. This line of reasoning suggests that key to tackle the unemployment problem during recessions is to restore the profitability of low-productivity firms, or in other words, subsidize matches with low-skilled workers. Policies restoring firms’ profitability are the most effective in economic downturns. Note that also the effectiveness of this latter type of policies is time varying. Specifically, subsidies to worker-employer low-productivity matches are much more effective during economic downturns than expansions. The time-varying effect of labor-market policies derives from the property that the impulse responses to aggregate shocks vary with the state of the economy. Since fiscal stimulus is typically timed during recessions, assessing the response of the labor market and output to aggregate shocks during recessions is of particular relevance. I show that in the model unemployment is more responsive to changes in aggregate conditions during recessions than during normal times.

State-dependent effects of aggregate shocks. As in Section 1.4.3, I derive the dynamic response to a positive aggregate shock by comparing the expected paths of two simulated economies. For the IRF in “bad times,” the first simulated economy starts with the level of the endogenous variable associated with the lowest aggregate state but is in the median state. The second simulated economy starts with the same level of the endogenous variable and is in the lowest aggregate state. The

difference between the two paths is the response over time to a positive impulse during recessions. For the IRF in “normal times” instead, the first economy starts with the level of the endogenous variable associated with the median state but is in the highest state. The second simulated economy starts with the same level of the endogenous variable and is in the median state. The difference between the two paths is the response over time to a positive impulse during normal times. In both cases, I report the average response across 5,000 replications. Critically, the size of the shock is the same across scenarios. In Figure 1.14, Panel A shows that the employment rate is much more responsive to aggregate shocks in bad than in good times. The difference in responsiveness between bad and good times is instead far less pronounced for output (see Figure 1.14, Panel B). This latter result is perhaps not surprising given that the behavior of output is far less asymmetric than that of the employment rate (see Sections 1.4.2 and 1.4.3). Overall, the analysis suggests that the model economy features *counter-cyclical* IRFs. This property comes entirely from the asymmetric amplification and propagation of symmetric shocks. In Appendix B.1, I document that the response of the actual U.S. unemployment rate to shocks is systematically stronger during recessions than during expansions. To this aim, I follow Bachmann et al. (2010) and estimate a two-stage time series model. In the first stage, I estimate an AR(2) process for the U.S. unemployment rate. Then in the second stage, I regress the absolute value of the first stage residuals on the average lagged unemployment rate to assess whether residual variance differs during recessions. The estimates show that the U.S. unemployment rate displays conditional heteroskedasticity. Specifically, the variance of the reduced-form innovations raise dramatically during times of high unemployment. However, this finding can be rationalized in two different ways: (1) the shocks driving fluctuations have constant variance but U.S. unemployment respond more to these shocks during recessions than during booms. Note that this is exactly what happens in the model. (2) Aggregate

shocks are conditionally heteroskedastic with recessions initiated by shocks of larger size. However, Berger and Vavra (2012) show that there is no evidence of conditional heteroskedasticity for shocks commonly used in the business cycle literature. These facts seem to suggest that asymmetric responses, instead of asymmetric shocks, are responsible for heteroskedasticity in the U.S. unemployment rate.

State-dependent effects of fiscal stimulus. To understand the exact link between aggregate shocks and exogenous changes in policies, I introduce into the model four tax instruments: (1) sales tax, τ_S ; (2) labor income tax, τ_W ; (3) payroll tax, τ_F ; and (4) recruiting costs expensing, τ_K . Equations (1.8) and (1.9) then become,

$$S_s(x) = \max \{S_s^c(x), 0\} \quad (1.27)$$

with

$$S_s^c(x) = (1 - \tau_S) z_s x - \left(\frac{1 + \tau_F}{1 - \tau_W} \right) \lambda + \beta \sum_{s'} \pi_{s,s'} [1 - \delta_{s'}(x) - \eta \phi_s(x)] S_{s'}(x),$$

where $\Pi_s(z_s x, \lambda, \tau_S, \tau_F, \tau_W) \equiv \left[(1 - \tau_S) z_s x - \left(\frac{1 + \tau_F}{1 - \tau_W} \right) \lambda \right]$ denote “profits” accruing to the worker-employer relationship. The free-entry condition for each submarket is

$$(1 - \tau_K) k(x) = \beta \rho_s(x) \sum_{s'} \pi_{s,s'} (1 - \eta) S_{s'}(x). \quad (1.28)$$

Note that: (i) all tax rates affect $S_s(x)$ through the profit term $\Pi_s(\cdot)$. As such cuts in either tax rate can be seen as “subsidies” to the worker-employer match. Moreover, since these subsidies act like changes in the aggregate shock, exogenous changes

in tax rates can be interpreted as shocks to the aggregate state of the economy. Hence, the dynamics in Figure 1.14 can also be interpreted as responses to exogenous decreases in either tax rate. This observation implies that fiscal policy is more effective during recessions than expansions. These predictions are consistent with the empirical evidence produced by Auerbach and Gorodnichenko (2012); (ii) the expensing rate of vacancy posting costs only enters the free-entry condition (1.28). As discussed in Section 1.3.4, changes in the cost to post a job vacancy are irrelevant for rationing unemployment. As such policies working through that margin are ineffective in reducing unemployment during recessions.

1.7 Conclusions

I develop a search-and-matching model with heterogeneous workers that accounts for the asymmetric fluctuations of the U.S. labor market and output. The fundamental property of the model is that recessions are initiated by a burst of job losses and job-finding rates display asymmetries over the cycle. The model generates realistic volatility in unemployment and job vacancies preserving a downward-sloping Beveridge curve. As a by-product, the model provides stark predictions for the time series of skill-specific unemployment rates. These predictions hold in CPS micro data once I sort workers by age and education. The model predicts that the effectiveness of macroeconomic policies varies over the business cycle. For instance, the effects of policies that restore the profitability of low-productivity matches are time varying: these policies are much more effective during economic downturns than expansions. That is, the economy features impulse responses that vary with the state of the economy.

1.8 Tables and Figures

This section contains the main tables and figures of the essay.

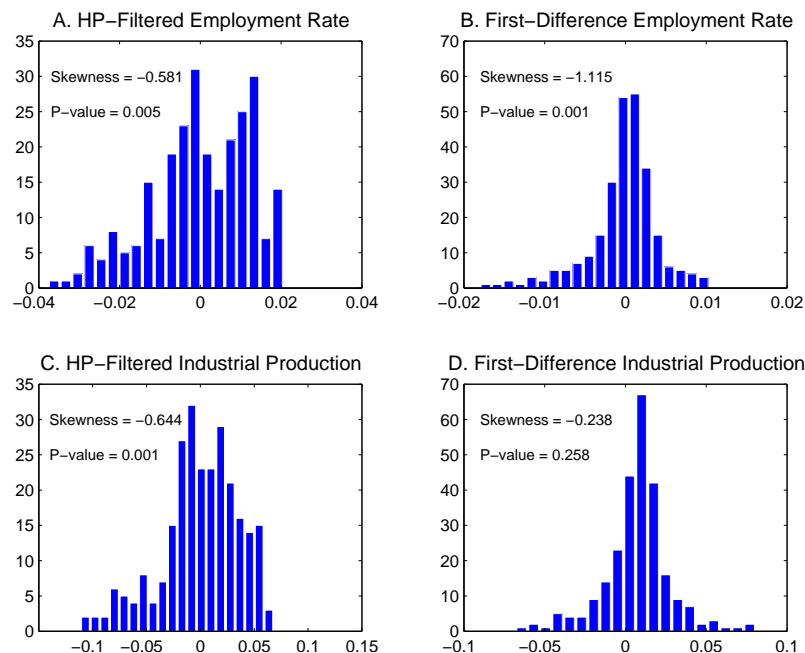


FIGURE 1.1: Asymmetry in Quarterly U.S. Data, 1948:Q1-2012:Q2

Notes: Panel A and B show the empirical distribution for respectively employment rate as deviations from the HP trend with smoothing parameter 10^5 and log-first-differences. Employment rate is the fraction of the labor force working in a given month, one minus the unemployment rate. The seasonally-adjusted unemployment rate is from the CPS survey of the BLS. Survey home page <http://www.bls.gov/cps/>. Panel C and D show the empirical distribution for respectively industrial production (IP) as deviations from the HP trend with smoothing parameter 1600 and log-first-differences. All data are logged quarterly averages of monthly series for the period 1948:Q1-2012:Q2. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>.

Table 1.1: Asymmetry in Quarterly U.S. Hours, 1948:Q1-2012:Q2

x	Hours	Hours per worker
$Skew(\tilde{x})$	-0.329** (0.028)	0.303 (0.165)
$Skew(\Delta x)$	-0.513** (0.036)	0.120 (0.225)

Notes: Hours and hours per worker are respectively seasonally-adjusted quarterly hours worked and hours to employment ratio in the nonfarm business sector. Data are from the LPC release of the BLS for the period 1948:Q1-2012:Q2. Release home page <http://www.bls.gov/lpc>. Nonfarm business sector output (NBSO) excludes from the business sector the farm sector. Business sector output (BSO) is the annual-weighted index constructed by the BLS after excluding from gross domestic product (GDP) the following outputs: general government, nonprofit institutions, paid employees of private households, and the rental value of owner-occupied dwellings. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Variables \tilde{x} are in logs as deviations from the HP trend with smoothing parameter 10^5 . P-values (one-sided test) in parenthesis. ** denote statistical significance at 5% level.

Table 1.2: Asymmetry in Employment Rate Counterfactuals

x	e^{us}	e^{se}	e^{jfr}	e^{jsr}
$Skew(\tilde{x})$	-0.581*** (0.005)	-0.682*** (0.001)	-0.792* (0.061)	-0.330*** (0.009)
$Skew(\Delta x)$	-1.115*** (0.001)	-0.999** (0.010)	0.328* (0.078)	-0.409*** (0.000)

Notes: e^{us} is a logged quarterly average of the U.S. employment rate (fraction of the labor force working in a given month, one minus the unemployment rate). The seasonally-adjusted unemployment rate is from the CPS survey of the BLS for the period 1948:Q1-2012:Q2. Survey home page <http://www.bls.gov/cps/>. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. $e^{se} = f/(s + f)$ is the counterfactual employment series under stochastic equilibrium. $e^{jfr} = f/(\bar{s} + f)$ is the counterfactual employment series with job-separation rate fixed at the sample average. $e^{jsr} = \bar{f}/(s + \bar{f})$ is the counterfactual employment series with the job-finding rate fixed at the sample average. \tilde{x} 's are in logs as deviations from the HP trend with smoothing parameter 10^5 . P-values (one-sided test) in parenthesis. ***, **, * denote statistical significance respectively at 1%, 5% and 10% level.

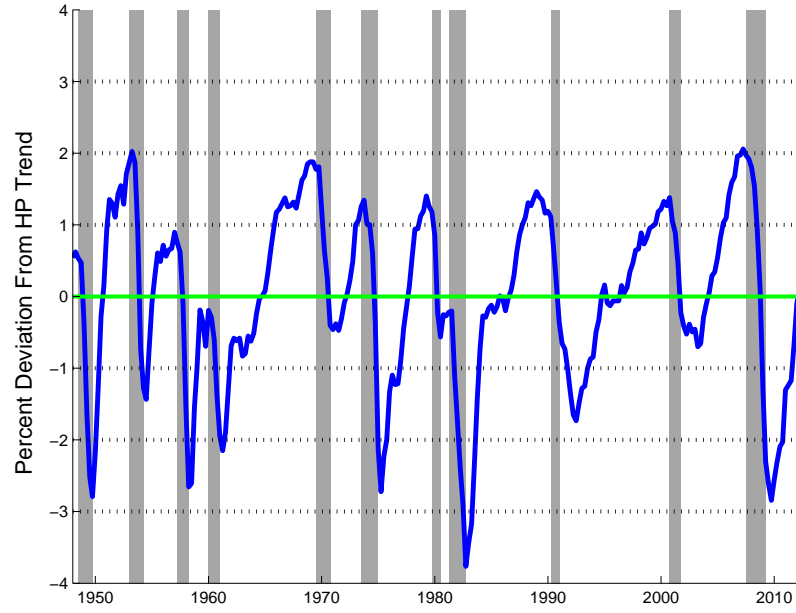


FIGURE 1.2: Detrended Quarterly U.S. Employment Rate, 1948:Q1-2012:Q2

Notes: Solid line shows a logged quarterly average of the U.S. employment rate (fraction of the labor force working in a given month, one minus the unemployment rate) as deviations from the HP trend with smoothing parameter 10^5 . Gray bands indicate NBER-dated recessions. The seasonally-adjusted unemployment rate is from the CPS survey of the BLS for the period 1948:Q1-2012:Q2. Survey home page <http://www.bls.gov/cps/>. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>.

Table 1.3: Calibration

	Interpretation	Value	Source/Target
β	<i>Discount factor</i>	0.9959	5% annual interest rate
δ	<i>Separation rate</i>	0.0189	JOLTS, 2001:M1-2011:M9
$k(x)$	<i>Vacancy cost</i>	$[0.0005, \dots, 0.3359]$	$\theta(x) = 1$ when $z = 1$
α	<i>Matching function: $m(v, u) = \mu v^\alpha u^{1-\alpha}$</i>	0.4	Brügemann (2008)
μ	<i>Matching function scale</i>	0.33	Median unemployment rate of 5.6%
η	<i>Worker Nash-bargaining weight</i>	0.6	Hosios (1990)'s condition
λ	<i>Flow value of unemployment</i>	0.44	62% of mean wage rate
$\ln(x_{180}/x_{20})$	<i>Productivity dispersion, 90-10 pctl range</i>	0.651	Syversen (2011)
ρ_z	<i>Autocorrelation of exogenous state</i>	0.9642	LPC, 1948:Q1-2011:Q3
σ_ϵ	<i>Standard deviation of shocks</i>	0.0055	LPC, 1948:Q1-2011:Q3

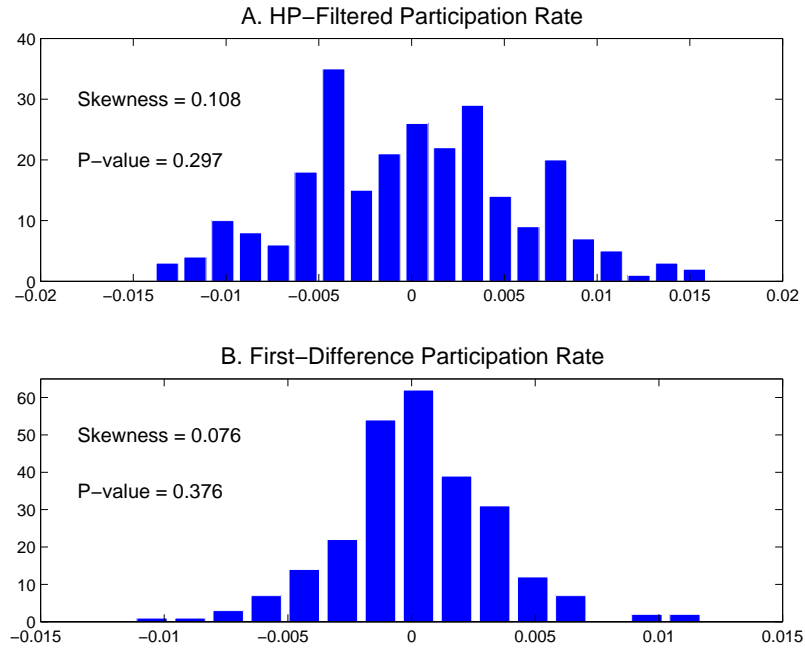


FIGURE 1.3: Asymmetry in Quarterly U.S. Participation Rate, 1948:Q1-2012:Q2

Notes: Panel A and B show the empirical distribution for respectively participation rate as deviations from the HP trend with smoothing parameter 10^5 and log-first-differences. Participation rate is the fraction of the population in the labor force (employed plus unemployed to population ratio). Data are from the CPS survey of the BLS. Survey home page <http://www.bls.gov/cps/>. Data are logged quarterly averages of monthly series for the period 1948:Q1-2012:Q2. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>.

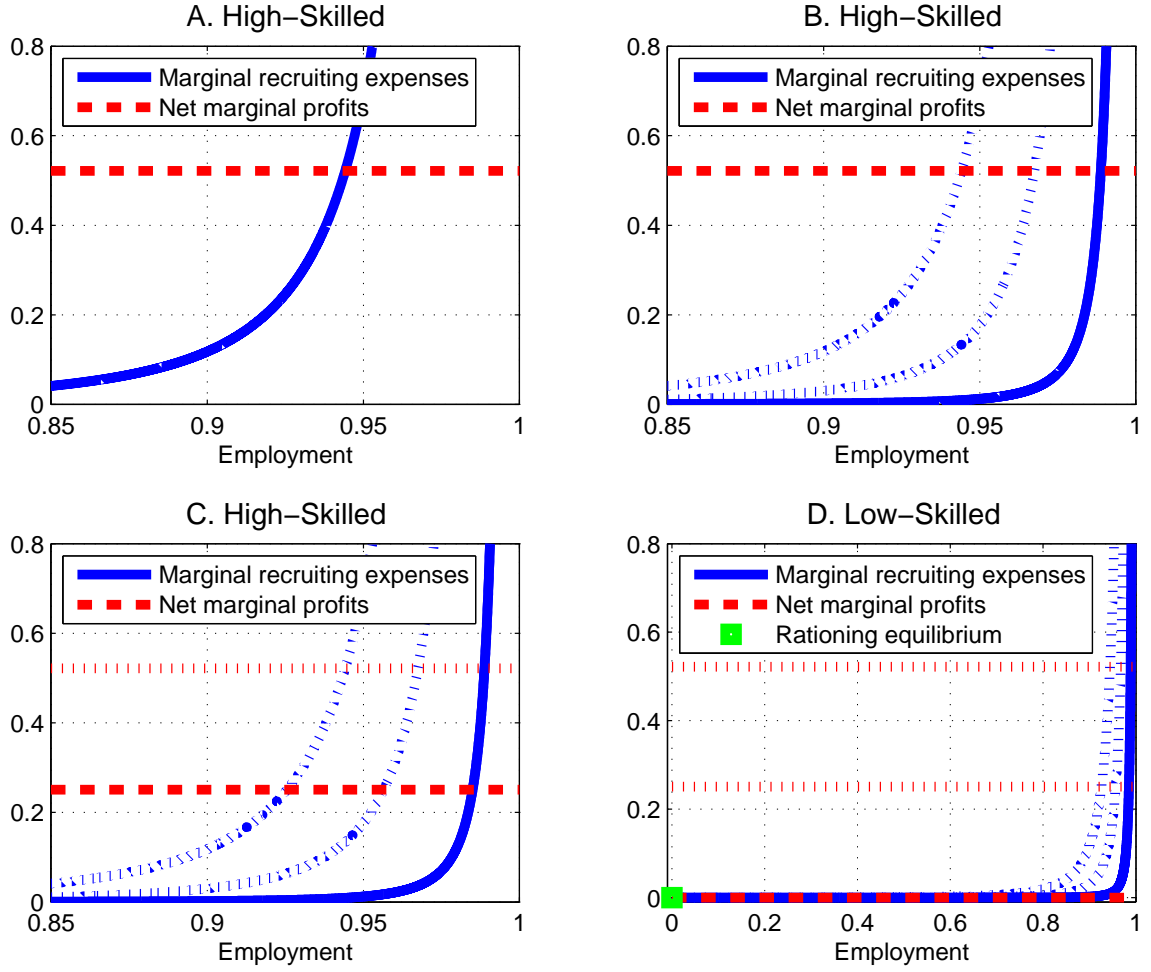


FIGURE 1.4: Frictional and Rationing Unemployment

Notes: On the x -axis, employment is steady-state employment as in equation (1.12). On the y -axis, net marginal profits and marginal recruiting expenses are respectively the left-hand and right-hand side of equation (1.13). High-skilled workers have productivity $p(x_{\text{high}}) > \hat{p}(x)$ and low-skilled workers have productivity $p(x_{\text{low}}) \leq \hat{p}(x)$. The threshold $\hat{p}(x)$ is determined by equation (1.11).

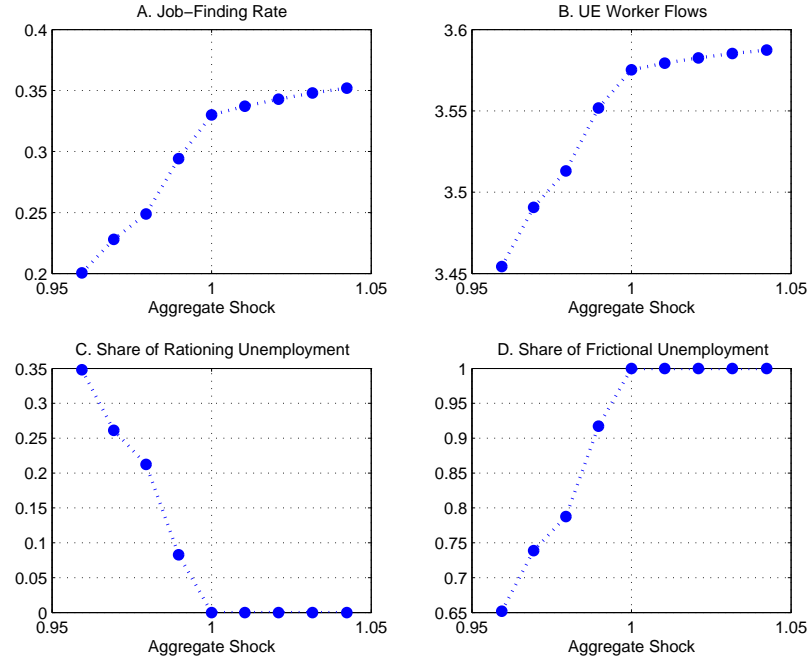


FIGURE 1.5: Aggregate Job-Finding Rate in the Segmented Labor Market

Notes: Panels A-D refer to the (segmented) labor market in steady state, i.e., $z_t = z_s$ for $s \in \{1, \dots, \mathcal{S}\}$ and all t . The x -axis is the support of the aggregate shock, z_s with $s \in \{1, 2, \dots, 9\}$. I compute the steady-state values of the endogenous variables numerically with the same parameter values used for the quantitative analysis of Section 1.4. Details of the calibration are in Section 3.6.1, Table 1.3.

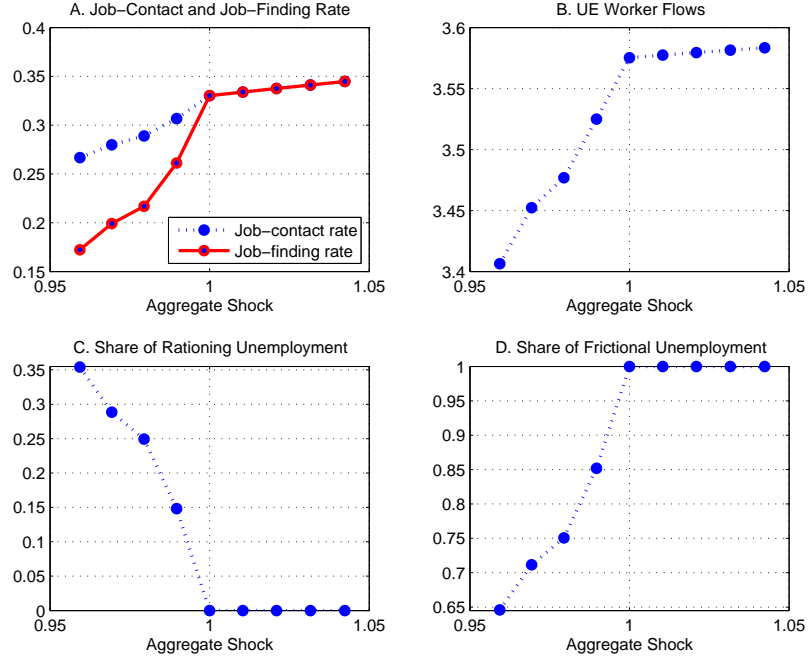


FIGURE 1.6: Aggregate Job-Finding Rate in the Integrated Labor Market

Notes: Panels A-D refer to the (integrated) labor market in steady state, i.e., $z_t = z_s$ for $s \in \{1, \dots, \mathcal{S}\}$ and all t . The x -axis is the support of the aggregate shock, z_s with $s \in \{1, 2, \dots, 9\}$. I compute the steady-state values of the endogenous variables numerically with calibrated parameter values. I set the vacancy cost to $k = 0.168$ such that $\theta_{s_m}(\pi_{s_m}) = 1$ ($z_{s_m} = 1$ in my normalization). The rest of the parameters are set as in Section 3.6.1, Table 1.3.

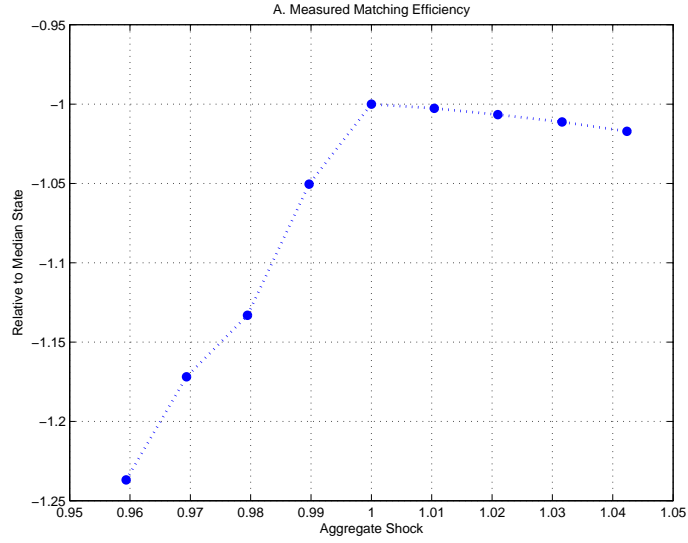


FIGURE 1.7: Time-Varying Matching Efficiency

Notes: Panel A refers to the labor market in steady state. The x -axis is the support of the aggregate shock, z_s with $s = \{1, 2, \dots, 9\}$. The y -axis is measured matching efficiency relative to the median state ($z_{s_m} = 1$ in my normalization), i.e., $\ln \tilde{A}_s / |\ln \tilde{A}_{s_m}|$. I compute the steady-state values of the endogenous variables numerically with the same parameter values used for the quantitative analysis of Section 1.4. Details of the calibration are in Section 3.6.1, Table 1.3.

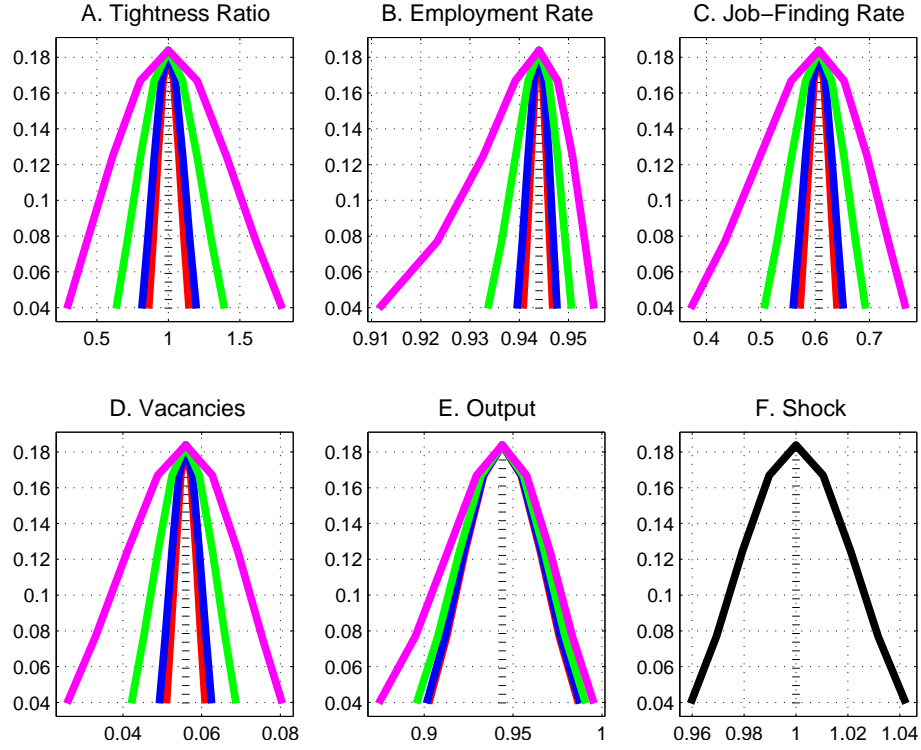


FIGURE 1.8: Probability Distributions under Stochastic Equilibrium

Notes: The unconditional probability distributions are computed numerically. Details of the calibration are in Table A.5 in Appendix B.1. Given the calibrated volatility of aggregate shocks, to induce greater dispersion in the distribution of the tightness ratio, I progressively increase the value of the worker's outside option, λ . Specifically, I use $\lambda = 73\%$ of mean wage (red line), $\lambda = 80\%$ of mean wage (blue line), $\lambda = 90\%$ of mean wage (green line), and $\lambda = 95\%$ of mean wage (purple line). In each case, I calibrate the cost of posting a vacancy to match the same median value of the variable over all λ 's calibration. The vertical dashed line denotes the median value of the variable.

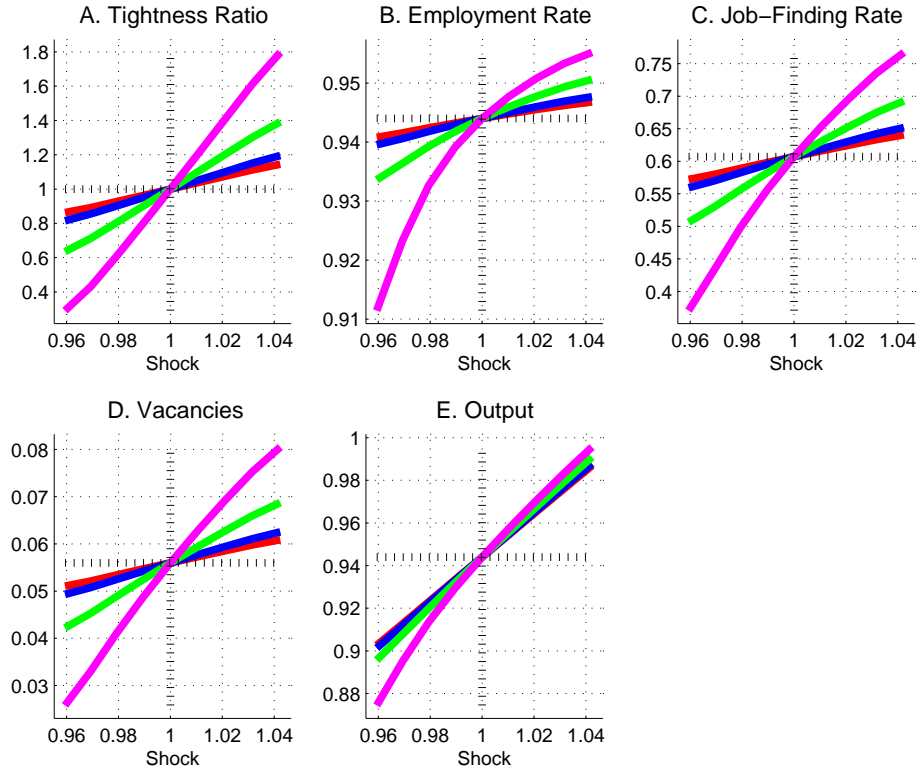


FIGURE 1.9: Policy Rules under Stochastic Equilibrium

Notes: Policy rules are computed numerically. Details of the calibration are in Table A.5 in Appendix B.1. Given the calibrated volatility of aggregate shocks, to induce greater dispersion in the distribution of the tightness ratio, I progressively increase the value of the worker's outside option, λ . Specifically, I use $\lambda = 73\%$ of mean wage (red line), $\lambda = 80\%$ of mean wage (blue line), $\lambda = 90\%$ of mean wage (green line), and $\lambda = 95\%$ of mean wage (purple line). In each case, I calibrate the cost of posting a vacancy to match the same median value of the variable over all λ 's calibration. The vertical dashed line denotes the median value of the aggregate shock.

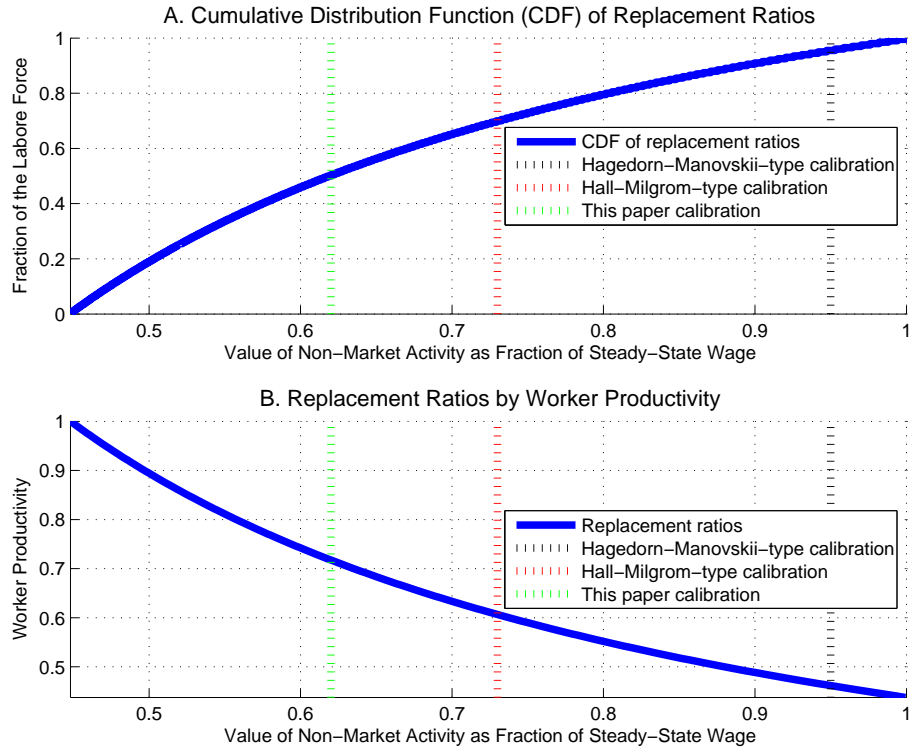


FIGURE 1.10: Replacement Ratios

Notes: In Hall-Milgrom-type calibration, the worker's outside option is set to 73% of steady-state wage. In Hagedorn-Manovskii-type calibration, the worker's outside option is set to 95% of steady-state wage. In this paper calibration, the worker's outside option is set to 62% of steady-state wage. See Table 1.3 for further details on calibration.

Table 1.4: Volatility and Comovement in U.S. Data and Model

	Data	EJD Model	CJD Model
A. Volatility			
$sd(u)/sd(p)$	9.851	9.051	1.327
$sd(\phi)/sd(p)$	7.213	9.232	1.491
$sd(v)/sd(p)$	8.131	5.464	2.484
B. Comovement			
$corr(u, v)$	-0.858	-0.561	-0.812
$corr(u, p)$	-0.412	-0.773	-0.912
$corr(\phi, p)$	0.383	0.824	0.996
$corr(v, p)$	0.433	0.830	0.978

Notes: The seasonally-adjusted unemployment rate, u , is from the CPS survey of the BLS. Survey home page <http://www.bls.gov/cps/>. Labor productivity, p , is seasonally-adjusted quarterly real output per worker in the nonfarm business sector constructed by the BLS from the LPC release. Release home page <http://www.bls.gov/lpc>. The series are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Job-finding rates, ϕ , are calculated based on Shimer (2012). Vacancies, v , are the composite Help-Wanted Index constructed by Barnichon (2010). The variables u , ϕ , and v are quarterly averages of monthly series. All series cover the period 1948:Q1-2011:Q3. EJD model refers to the version of the model with endogenous job destruction. CJD model refer to the version of the model with constant and exogenous job destruction. Model simulated data are quarterly averages of 765 observations at the monthly frequency. The statistics reported are averages across 500 replications. All variables are reported in logs as deviations from the HP trend with smoothing parameter 10^5 .

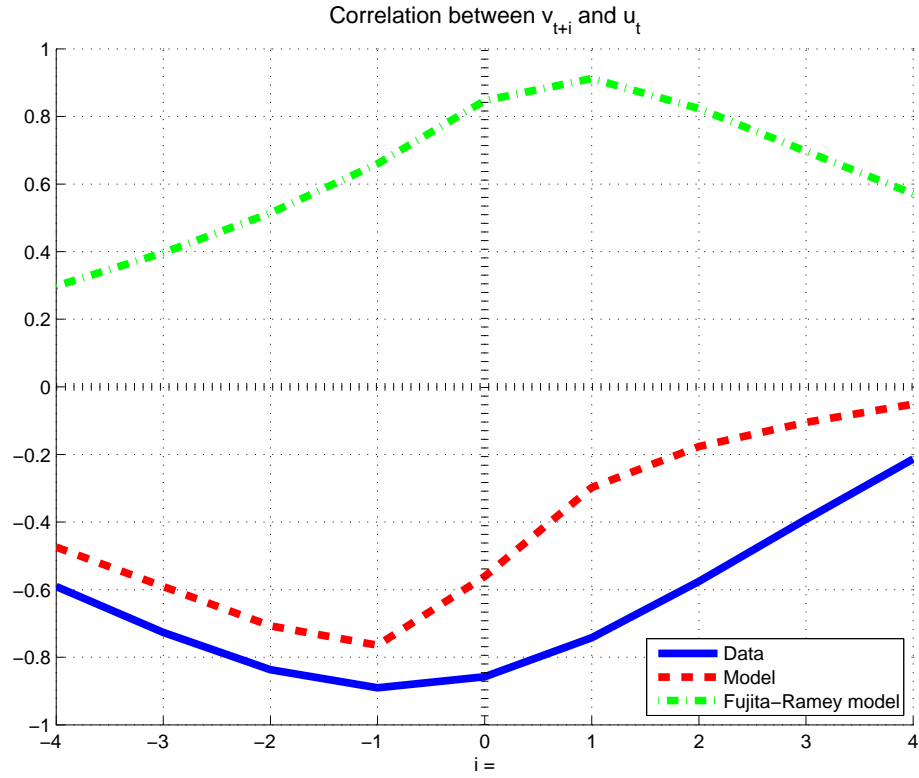


FIGURE 1.11: Dynamic Beveridge Curve

Notes: The seasonally-adjusted unemployment rate, u , is from the CPS survey of the BLS. Survey home page <http://www.bls.gov/cps/>. The series is downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Vacancies, v , are the composite Help-Wanted Index constructed by Barnichon (2010). Model refers to the EJD model. Fujita-Ramey model refers to a stochastic discrete-time version of Mortensen and Pissarides (1994)'s model as presented in Fujita and Ramey (2012).

Table 1.5: Asymmetry in U.S. Data and Model

	Data	EJD Model	CJD Model
A. Skewness in levels			
$Skew(e)$	-0.591***	-0.877	-0.254
$Skew(y)$	-0.345**	-0.246	-0.013
$Skew(\phi)$	-0.249**	-0.922	-0.230
$Skew(v)$	-0.469***	0.542	-0.130
B. Skewness in growth rates			
$Skew(\Delta e)$	-0.958***	-0.794	0.044
$Skew(\Delta y)$	-0.170	-0.055	0.005
$Skew(\Delta \phi)$	0.235***	0.319	0.042
$Skew(\Delta v)$	-0.613***	0.670	0.044

Notes: The seasonally-adjusted unemployment rate, u , is from the CPS survey of the BLS. Survey home page <http://www.bls.gov/cps/>. Labor productivity, p , is seasonally-adjusted quarterly real output per worker in the nonfarm business sector constructed by the BLS from the LPC release. Release home page <http://www.bls.gov/lpc>. Output, y , is industrial production (IP). The series are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Job-finding rates, ϕ , are calculated based on Shimer (2012). Vacancies, v , are the composite Help-Wanted Index constructed by Barnichon (2010). The variables u , ϕ , and v are quarterly averages of monthly series. All series cover the period 1948:Q1-2011:Q3. EJD model refers to the version of the model with endogenous job destruction. CJD model refers to the version of the model with constant and exogenous job destruction. Model simulated data are quarterly averages of 765 observations at the monthly frequency. The statistics reported are averages across 500 replications. In Panel A, all variables are reported in logs as deviations from the HP trend with smoothing parameter 10^5 . In Panel B, all variables are reported as 3-months log-differences. ***, ** denote statistical significance at respectively 1% and 5% level.

Table 1.6: Asymmetry in Employment Rate Counterfactuals

x	e	e^{jfr}	e^{jsr}
A. Skewness in levels			
$Skew(x)$	-0.877	-0.937	-1.068
B. Skewness in growth rates			
$Skew(\Delta x)$	-0.794	1.691	-1.075

Notes: EJD model simulations are 765 observations at the monthly frequency. The statistics reported are averages across 500 replications. e is the employment rate (fraction of the labor force working in a given month, one minus the unemployment rate), e^{jfr} is the counterfactual employment rate series with job-separation rates fixed at steady-state values, and e^{jsr} is the counterfactual employment rate series with job-finding rates fixed at steady-state values. In Panel A, all variables are quarterly averages reported in logs as deviations from the HP trend with smoothing parameter 10^5 . In Panel B, all variables are reported as 3-months log-differences.

Table 1.7: Asymmetry in Output and its Components

	Y	z	$\sum_x x e(x)$
A. Skewness in levels	-0.246	0.000	-0.858
B. Skewness in growth rates	-0.055	0.000	-0.761

Notes: EJD model simulated data are quarterly averages of 765 observations at the monthly frequency. The statistics reported are averages across 500 replications. In Panel A, all variables are reported in logs as deviations from the HP trend with smoothing parameter 10^5 . In Panel B, all variables are reported as 3-months log-differences.

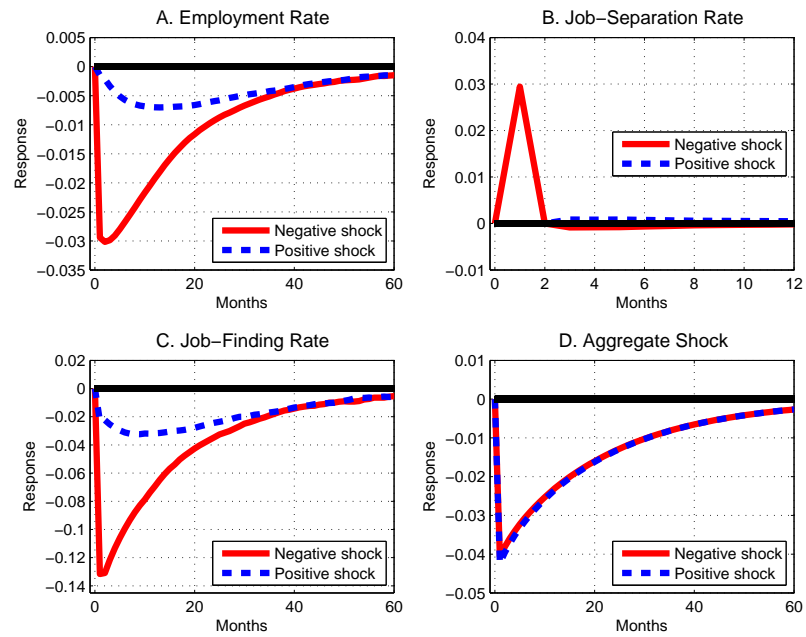


FIGURE 1.12: Impulse Responses for the Employment Rate

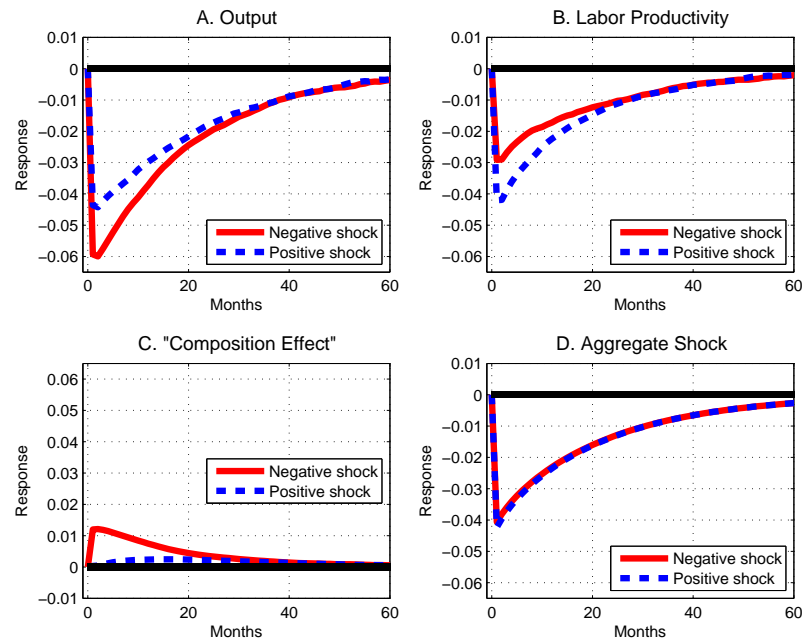


FIGURE 1.13: Impulse Responses for Output

Table 1.8: Average Unemployment Rates by Age Group

	16-24	25-34	35-44	45-54	55-64	65+
Percent (%)	12.90	6.16	4.64	4.09	3.92	3.83
Normalized	3.16	1.51	1.14	1	0.96	0.94

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. Seasonal adjustment is implemented with a 13-term symmetric moving average. The first row shows sample averages of unemployment rates by age in percent. The second row reports average unemployment rates by age relative to that of 45-54 years old.

Table 1.9: Average Unemployment Rates by Education Group

	HSD	HSG/SC	CGPC
Percent (%)	12.72	6.10	2.79
Normalized	4.56	2.19	1

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. HSD, HSG, SC and CGPC denote respectively High School Dropouts, High School Graduates, Some College and College Graduates and Post-College Degree Holders. Seasonal adjustment is implemented with a 13-term symmetric moving average. The first row shows sample averages of unemployment rates by education in percent. The second row reports average unemployment rates by education relative to that of CGPC.

Table 1.10: Average Unemployment Rates by Age/Education Group

	16-24	25-34	35-44	45-54	55-64	65+
A. Average, percent (%)						
HSD	21.01	13.02	9.46	7.57	6.08	5.08
HSG/SC	10.45	6.60	4.87	4.09	3.82	3.77
CGPC	6.05	2.86	2.35	2.31	2.50	2.71
B. Average, normalized						
HSD	9.10	5.64	4.10	3.28	2.63	2.20
HSG/SC	4.52	2.86	2.11	1.77	1.65	1.63
CGPC	2.62	1.24	1.02	1	1.08	1.17

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. HSD, HSG, SC and CGPC denote respectively High School Dropouts, High School Graduates, Some College and College Graduates and Post-College Degree Holders. Seasonal adjustment is implemented with a 13-term symmetric moving average. Panel A shows sample averages of unemployment rates by age/education group. Panel B shows average unemployment rates by age/education relative to that of 45-54 years old with CGPC education level.

Table 1.11: Average Unemployment Rates by Skill/Productivity Group

	x_1	x_2	x_3	x_4	x_5	x_6	$(x_7 - x_{200})$
Percent (%)	46.35	34.15	32.94	21.17	12.92	12.13	5.43
Normalized	8.54	6.29	6.07	3.90	2.38	2.23	1

Notes: The statistics reported are average unemployment rates in the EJD model by skill/productivity group averaged across 500 replications. The second row reports average unemployment rates by skill/productivity relative to that of $(x_7 - x_{200})$ group.

Table 1.12: Volatility Shares by Age Group

	16-24	25-34	35-44	45-54	55-64	65+
Cov. not incl.	47.28	25.93	8.87	12.32	5.28	0.33
Cov. incl.	30.84	28.12	13.53	14.69	10.86	1.96

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. Seasonal adjustment is implemented with a 13-term symmetric moving average. The statistics reported are percentage shares of total U.S. unemployment rate variance attributed to each age group. "Cov. not incl." means covariance terms are ignored such that total variation is the sum of the variables' variances. "Cov. incl." means total variation includes covariance terms such that total variation is the sum of the variables' variances plus two times their covariance.

Table 1.13: Volatility Shares by Education Group

	HSD	HSG/SC	CGPC
Cov. not incl.	24.51	69.39	6.10
Cov. incl.	26.16	62.38	11.46

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. HSD, HSG, SC and CGPC denote respectively High School Dropouts, High School Graduates, Some College and College Graduates and Post-College Degree Holders. Seasonal adjustment is implemented with a 13-term symmetric moving average. The statistics reported are percentage shares of total U.S. unemployment rate variance attributed to each education group. “Cov. not incl.” means covariance terms are ignored such that total variation is the sum of the variables’ variances. “Cov. incl.” means total variation includes covariance terms such that total variation is the sum of the variables’ variances plus two times their covariance.

Table 1.14: Volatility Shares by Skill/Productivity Group

	x_1	x_2	x_3	x_4	x_5	x_6	$x_7 - x_{200}$
Cov. not incl.	29.19	22.81	22.03	13.67	6.34	5.83	0.13
Cov. incl.	19.72	19.37	18.96	13.56	7.79	7.16	13.44

Notes: The statistics reported are percentage shares of total unemployment rate variance in the EJD model attributed to each skill/productivity group averaged across 500 replications. “Cov. not incl.” means covariance terms are ignored such that total variation is the sum of the variables’ variances. “Cov. incl.” means total variation includes covariance terms such that total variation is the sum of the variables’ variances plus two times their covariance.

Table 1.15: Unemployment versus Participation Margin by Age Group

	16-24	25-34	35-44	45-54	55-64	65+
Unemployment margin	98.95	99.81	99.81	99.82	99.67	97.12
Participation margin	1.05	0.19	0.19	0.18	0.33	2.88

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. Seasonal adjustment is implemented with a 13-term symmetric moving average. Data are logged and HP-filtered with smoothing parameter 129,000 at the monthly frequency. See Ravn and Uhlig (2002) for a thorough discussion on the choice of the HP smoothing parameter and the frequency of observations. The statistics reported are percentage shares of total unemployment shares variance attributed to the unemployment and participation margin by each age group. Covariance terms are not included such that total variation is the sum of the variables' variances.

Table 1.16: Unemployment versus Participation Margin by Education Group

	HSD	HSG/SC	CGPC
Unemployment margin	98.78	99.94	99.80
Participation margin	1.22	0.06	0.20

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. HSD, HSG, SC and CGPC denote respectively High School Dropouts, High School Graduates, Some College and College Graduates and Post-College Degree Holders. Seasonal adjustment is implemented with a 13-term symmetric moving average. Data are logged and HP-filtered with smoothing parameter 129,000 at the monthly frequency. See Ravn and Uhlig (2002) for a thorough discussion on the choice of the HP smoothing parameter and the frequency of observations. The statistics reported are percentage shares of total unemployment shares variance attributed to the unemployment and participation margin by each education group. Covariance terms are not included such that total variation is the sum of the variables' variances.

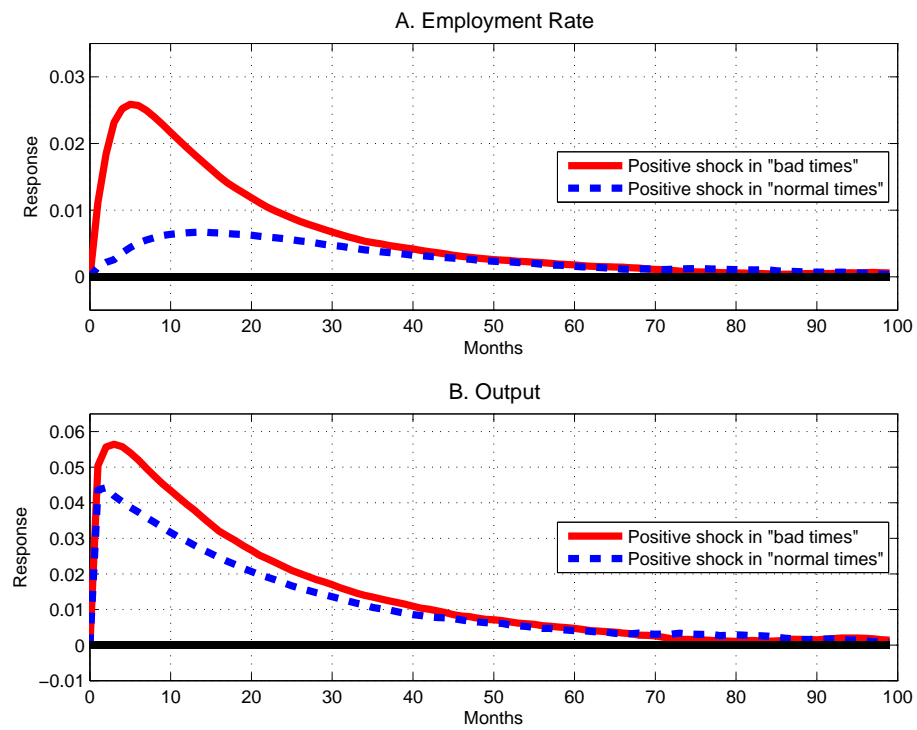


FIGURE 1.14: State-Dependent Impulse Responses

Volatility and Slow Technology Diffusion: The Case of Information Technologies

2.1 Introduction

Does business cycle volatility affect long-run economic growth?¹ Despite the number of empirical and theoretical studies, a definitive answer to this question is still missing. In a seminal paper, Ramey and Ramey (1995) document a strong empirical negative relationship between volatility and economic growth. Subsequently, Martin and Rogers (1997), Acemoglu et al. (2003), Aghion et al. (2009), Aghion et al. (2010) and Posch (2011) confirm this negative link for other data sources, samples and estimation strategies. Compared to Ramey and Ramey, these papers add alternative controls such as exchange rate variability, financial development, measures of openness, institutions, monetary and fiscal policy variability. However, there are

¹ This simple question is undoubtedly one of the most relevant in modern macroeconomics. Using the forceful words of Lucas (1987), “*the welfare consequences of ‘small’ changes [in growth rates] are enormous,*” and even more strikingly, Lucas’s calculations imply that “*consumers would require a 20 per cent across-the-board consumption increase to accept voluntarily a reduction in the consumption growth rate from 0.03 to 0.02, and would surrender 42 per cent across the board to obtain an increase in the growth rate from 0.03 to 0.06.*” These provocative results make clear that business fluctuations can potentially entail large welfare losses if they adversely affect long-run economic growth.

several studies suggesting that the volatility-growth link is not as strong when using the time series dimension of the data.²

The aforementioned literature has invariably focused on average growth rates of real output and/or labor productivity as a measure of long-run economic performance. This paper tackles the same research question from a different angle. We consider direct measures of technology diffusion and ask a new question: *does business cycle volatility affect the rate at which new technologies are adopted?* We believe the answer to this question provides new insights on the link between volatility, total factor productivity (TFP), long-run economic growth and cross-country differences in incomes per capita. Our specific focus on technology diffusion as opposed to output-related growth measures is also motivated by the striking fact presented by Klenow and Rodríguez-Clare (1997) that most of the world income inequality is due to cross-country differences in TFP. Therefore, in order to understand why some countries are richer than others we need to understand why they are more productive. Arguably, an important determinant of these cross-country differences in TFP is technology.

We demonstrate the existence of a strong negative relationship between volatility and technology diffusion. We consider three major information and communication technologies (ICT's)—personal computers, internet and cell phones per capita—and show that more volatile countries take more time to adopt newly available technologies.

The positive relationship between volatility and time diffusion lags is rather robust and holds after controlling for cross-country differences in growth rates of real

² For pooled OLS regressions, Grier and Tullock (1989) find a positive relationship between volatility and growth after controlling for inflation variability and other standard control variables (human capital, initial capital, etc.). The standard deviation of inflation enters the growth equation with a negative sign. For fixed-effects regressions, Imbs (2007) and Chong and Gradstein (2009) find an ambiguous effect—either a positive or negative link—at the industry/sectoral level. Ramey and Ramey (1995) also run fixed-effects panel data regressions and confirm their cross-country finding of a negative relationship between volatility and growth.

GDP per capita. Controlling for output growth is important for two reasons. First, it allows us to interpret the results in the spirit of a mean-preserving spread across countries. Countries whose growth rates are relatively riskier are also farther behind the technological frontier. Second, by having average growth rates as a covariate in the regression analysis, we indirectly control for all the variables that the vast literature on cross-country growth regressions has shown to be relevant for growth. This latter feature is particularly appealing given the large set of variables that the growth literature has found to be important for economic growth.

To interpret the empirical findings, we offer a simple stochastic model of technology adoption in which we derive in closed form the theoretical mapping between time diffusion lags, growth and volatility. In the model, adoption of a new technology is irreversible. The problem faced by a firm is equivalent to exercising a call option. Uncertainty about the future generates a real value of inaction which delays the adoption of the new frontier technology. In this respect, the model is a real options theory of technology adoption.

The model provides the stark prediction that uncertainty acts as a *barrier* to technology adoption. Hence, it renders a new interpretation to the long-run effects of volatility. Specifically, relatively small differences in uncertainty are consistent with large variation in time diffusion lags. In this latter regard, our results are reminiscent of the findings in Parente and Prescott (1994). In a *deterministic* environment whose theoretical structure is completely different than ours, they argue that relatively small differences in barriers to technology adoption are needed to account for the large cross-country differences in incomes per capita.

The model also accounts for the negative relationship between average growth rates and diffusion lags which we observe in the data for a large cross-section of developing and developed countries. Furthermore, it predicts the cross-country dispersion in incomes per capita to increase at the onset of a technological revolution. This

is indeed what we document has happened over the so-called IT revolution: *major technological innovations act like shocks to the cross-country distribution of incomes per capita.*

Our empirical findings strengthen the empirical evidence on the negative link between volatility and economic growth presented by Ramey and Ramey (1995). Ramey and Ramey show that volatility and growth are negatively correlated after controlling for a set of variables identified by Levine and Renelt (1992) as important control variables for cross-country growth regressions (e.g., human capital, initial physical capital, etc.). The negative relationship is statistically significant at the 5 percent level in a sample of 92 developing and developed countries for 1962-1985. In the OECD sample for 1952-1988, the negative sign is significantly different than zero at the 10 percent level. The negative relationship persists and remains almost unchanged after controlling for the investment/GDP ratio. This latter result is important for our purpose because it rules out investment in physical capital and propensity to save as the main channel through which uncertainty affects growth and potentially opens the avenue to what Parente and Prescott (1994) name “*technology adoption investment.*”³

Following Ramey and Ramey (1995), we measure country-specific growth and volatility as respectively mean and standard deviation of per-capita real GDP annual growth rates. We then construct *time adoption lags* for the three ICT’s following the approach developed by Comin et al. (2008). Specifically, for each technology the time adoption lag measures how many years ago the U.S. last had the usage level that any other country had in a benchmark year, the 2002 in the base case. These time lags give us a measure of the distance from the technological frontier,

³ A negative relationship between volatility and growth emerges in models with investment irreversibilities and fixed costs of adjustment (e.g., Bernanke (1983), Pindyck (1991), Dixit and Pindyck (2008)). Aizenman (1993) develop a model with investment irreversibilities in which a rise in policy uncertainty leads to reduced growth.

the U.S. in our case. We find that more volatile countries are farther behind the U.S., that is, they have higher time adoption lags. This finding holds even after controlling for cross-country differences in average growth rates, suggesting that besides the volatility-growth relationship previously documented by Ramey and Ramey (1995), there is also an independent link between volatility and technology adoption. Switching the focus from output growth to technology adoption helps to highlight a specific mechanism, namely technology adoption, through which fluctuations in economic activity might affect the medium and long-run performance of a country. Output growth is affected by several factors such as growth in factors of production, efficiency gains due to reallocation of resources and by technological improvements. However, each of these determinants would arguably call for a different transmission mechanism, and certainly have different implications for the medium and long-run. By focusing on the process of technology diffusion, we do not intend to deny the importance of either factor accumulation or efficient reallocation of resources, but we try to explore one potential mechanism we think it is particularly relevant for the long-run performance of a country.

A potential criticism to the time adoption lags a la' Comin et al. (2008), particularly relevant for the focus of our paper, is that their computation relies exclusively on the usage level of a specific technology without adjusting for the quality improvements that technologies experience over time. If a poor country is catching up with the technological leader, it could be installing better quality machines such that the time adoption lags would over-estimate the actual distance from the frontier.

To address this issue, we propose a new measure of *quality-adjusted time adoption lags*, and show that the negative link between volatility and technology diffusion still holds. The new measure of time adoption lags is based on the observation that personal computers are technology-embodiment capital goods, and on the assumption that the frontier technology grows at a constant rate. We take the growth rate of the

frontier technology as the mean growth rate of the U.S. investment-specific technical change for 1973-2002. Following Greenwood et al. (1997), we calculate investment-specific technical change as the ratio of the implicit price deflator for nondurable consumption goods to the price index for computers and peripheral equipment.

As in the previous empirical studies on volatility and growth, we also find that the evidence is not as strong when we look at the time-series dimension of the data. After controlling for country-fixed effects, we find that periods of high volatility are associated with periods of relatively low investment in personal computers. The same negative relationship, even though not statistically significant, holds for the number of internet users per capita in a subsample of OECD countries. However, the sign of the relationship is reversed when we consider cell phones per capita. We conjecture that the lack of clear-cut evidence in the time-series is due to: 1) the time-series dimension of the data is far too short to provide reliable parameter estimates; and 2) the country dummies in the Fixed-Effects model absorb most of the variation of the data, making relatively hard the identification of the parameters of interest. In other words, being a “volatile country” is a very persistent characteristic such that the cross-country variation in the data overwhelms the time-series dimension.

Besides the link between volatility and technology diffusion, we also present new facts on the joint behavior of technology diffusion and incomes per capita. We document that the joint distribution of time adoption lags and per capita incomes is twin-peaked. While the “twin-peakedness” of the distribution of international incomes per capita is now well-known, to our knowledge, the twin-peakedness of the joint distribution of per capita incomes and time adoption lags is a new fact.⁴ Countries are polarized in two clubs: rich countries close to the technological frontier—low time adoption lags—versus poor countries lagging farther behind—high time

⁴ In a series of papers, Quah (1993, 1996a,b,c, 1997) show for the first time that international incomes per capita have a bimodal distribution, introducing in the literature the notions of clubs convergence, polarization and stratification of the world income distribution.

adoption lags. Importantly, these empirical observations reassure us that the time diffusion lags we consider are a sensible measure of long-run economic performance.

The rest of the paper is organized as follows. Section 3.2 presents a stochastic model of technology adoption, in which we derive in closed form the theoretical mapping between time adoption lags, growth and volatility. Section 2.3 contains a detailed description of the variables used in the empirical analysis. Section 2.4 provides evidence on the link between technology adoption and the cross-country distribution of incomes per capita. Section 2.5 presents the empirical evidence on volatility and technology adoption. Section 3.7 concludes.

2.2 A Real Options Model of Technology Adoption

In this section, we propose a simple stochastic model of technology adoption. To make the analysis as transparent as possible, we model a once-and-for-all adoption of a new technology, which unexpectedly becomes available to the world and dominates the current technology in use. We think of this scenario as the arrival of a General Purpose Technology (GPT), and the transition from the old to the new technology as a “technological revolution.”⁵

Specifically, we consider an economy in which a representative agent is initially endowed with a technology which produces a constant flow of output, i.e., there is no long-run growth. Unexpectedly, a new technology which ensures positive trend growth becomes available for adoption. The agent has then the option to adopt the new technology by paying a cost. Adoption is irreversible. After adoption, the frontier technology *in use* and so the income of the representative agent follows an exogenous stochastic process, which we model as a geometric Brownian motion with a

⁵ The literature refers to the transition period from the arrival of a GPT to the adoption as a “technological revolution.” Greenwood and Jovanovic (1999), Jovanovic and Rousseau (2003) and Pastor and Veronesi (2009) focus on the asset pricing implications of technological revolutions. Helpman (1998) and Jovanovic and Rousseau (2005) contain an extensive treatment of GPT’s.

certain drift and volatility. In the real world, drift and volatility of the income process after adoption depend not only upon the fundamental properties of the technology in use but also on all the other features of the economic environment. While of great relevance, in this paper we do not model explicitly the mechanisms through which the economic environment affects the drift and volatility of the income process. Real world countries largely differ by geography, institutions and governments' policies. We think of these differences as determinants of both the mean and volatility of the income process after adoption, and so affecting the profitability of adopting the frontier technology.

In our model economy, the representative agent trades off the benefits of adopting a better technology with the cost of adopting it, internalizing that adoption is an irreversible decision. Not surprisingly, higher adoption costs curtail the incentives to adopt, and an increase in the drift of the income process after adoption instead leans towards early adoption. Furthermore, the interaction of uncertainty with sunk costs of adoption creates a real value of inaction which delays the adoption of the frontier technology. To this latter respect, the model is a real options theory of technology adoption.

The ultimate goal of this section is twofold: first, to show that the notion of *time adoption lag*, which we use below in the empirical part of the paper, naturally raises from a model of technology adoption; second, to derive the theoretical mapping between time adoption lags, growth and volatility. The model provides a closed-form solution for the time adoption lag as a function of the adoption cost, interest rate, drift and volatility of the income process after adoption. The theoretical results of this section inform the empirical analysis of Section 2.4 and 2.5.

2.2.1 Economic Environment

Time is continuous, and runs forever as indexed by $t \geq 0$. A representative agent has linear preferences over consumption, $\mathcal{U}(C) = C$, and discount future payoffs by a factor $\delta \in (0, 1)$. Each period, the representative agent consumes all her income, and solves the following maximization problem:⁶

$$\max_{\{C(t)\}_{t=0}^{\infty}} \mathbb{E}_0 \left[\int_0^{\infty} e^{-\delta t} \mathcal{U}[C(t)] dt \right] \quad \text{subject to} \quad C(t) + \mathcal{A}C(T) = Y(t), \quad (2.1)$$

where \mathbb{E}_0 is the expectation conditional on information available at time $t = 0$. Output $Y(t)$ is produced according to the linear production function $Y(t) = \tilde{A}(t)$, where $\tilde{A}(t)$ denotes *technology in use* at time t . The representative agent receives $\tilde{A}(t)$ as an endowment. The installed technology is constant over time $\tilde{A}(t) = \tilde{A}_0$, such that it produces a constant flow of output. Unexpectedly at time $t = 0$, a new technology is invented. Let $A(t)$ denote this new *frontier technology*, which is available for adoption for all $t \geq 0$. After adoption, the dynamics of the frontier technology $A(t)$ and so the income process $Y(t)$ follows a geometric Brownian motion with drift μ_A and volatility σ_A :

$$dA = \mu_A A dt + \sigma_A A dW_A,$$

where $dW_A \stackrel{iid}{\sim} \mathcal{N}(0, dt)$ is a Wiener process, and the initial condition is normalized without loss of generality to $A_0 = 1$. To make the notation more transparent,

⁶ An interpretation of linear preferences in consumption is that the representative agent is risk neutral, such that she does not smooth consumption and each period consumes all her income. An alternative way to justify the linearity assumption without invoking risk-neutrality, is to consider a model economy with complete markets in which the agent perfectly insure her consumption risk against any idiosyncratic income risk through a set of state-contingent Arrow-Debreu securities. In this model economy, the agent maximizes expected utility by first maximizing income, and then choosing any consumption stream that costs less than income. Therefore, it is legitimate to study expected income maximizing behavior abstracting from consumption decisions.

we suppress time indices, for example, $dA \equiv dA(t)$, etc., unless needed for clarity. We model the process of technology adoption in an extremely simple fashion: the representative agent has to choose when to upgrade its technology, that is, the time at which she switches from the technology currently in use $\tilde{A}(t)$ to the new available frontier technology $A(t)$. Given the resource constraint $C(t) + \mathcal{AC}(T) = Y(t)$ for all $t \geq 0$, the value for the representative agent is the expected discounted value of output net of the adoption costs $\mathcal{AC}(t)$ incurred when the representative agent upgrades its technology:

$$V(\tilde{A}_0, A_0) = \max_{\gamma \in \Gamma} \mathbb{E}_0 \left[\int_0^T e^{-\delta t} Y(t) dt + \int_T^\infty e^{-\delta t} Y(t) dt - e^{-\delta T} \mathcal{AC}(T) \right], \quad (2.2)$$

where \tilde{A}_0 and A_0 are respectively the initial conditions for the installed and frontier technology. The maximization problem (2.2) is an *optimal stopping problem*, in which $\gamma = \{T, A(T)\}$ is the associated *impulse control policy*. The agent chooses a stopping time T , that is, the date at which she upgrades its technology switching from \tilde{A}_0 to $A(T)$. Hence, output is $Y(t) = \tilde{A}(t) = \tilde{A}_0$ for $0 \leq t < T$, and $Y(t) = A(t)$ for all $t \geq T$. The adoption cost as fraction of output $\mathcal{AC}(T) = cA(T)$ with $c \in (0, 1)$ is incurred at time T . Modeling the adoption costs as a fraction of output ensures that the costs do not become negligible relative to the level at which the new technology is adopted.

2.2.2 Time Adoption Lag

To solve problem (2.2), let $Z(t)$ denote the difference between the technology in use and the frontier technology—*latent technology*—for $t < T$,

$$Z \equiv \ln(\tilde{A}) - \ln(A) = Z_0 + \mu_Z t + \sigma_Z W_Z \quad (2.3)$$

where $Z_0 = \ln(\tilde{A}_0) - \ln(A_0)$ is the initial condition, $\mu_Z = \frac{1}{2}\sigma_A^2 - \mu_A$ is the drift, and $\sigma_Z = -\sigma_A$ is the volatility of the latent technology Z , with $W_Z \equiv W_A$. For $t \geq T$,

$$Z \equiv \ln(A) - \ln(A) = 0.$$

The latent technology Z is the relevant state variable of the optimal stopping problem (2.2). As we see below in more detail, the optimal adoption rule is a *threshold policy*: adopt the frontier technology as soon as the latent technology Z hits a threshold and operate it forever for all $t \geq T$.

We rewrite problem (2.2) in terms of the variable Z , such that:

$$V(Z_0, A_0) = \max_{b \leq Z} \mathbb{E}_0 \left\{ \int_0^{T(b)} e^{-\delta t} Y(t) dt + \int_{T(b)}^{\infty} e^{-\delta t} Y(t) dt - e^{-\delta T} \mathcal{AC}[T(b)] \right\}, \quad (2.4)$$

where Z_0 and A_0 are respectively the initial conditions for the latent and frontier technology. Output is now $Y(t) = e^{Z(t)} A(t) = \tilde{A}(t)$ for $t < T$, and $Y(t) = e^{Z(t)} A(t) = A(t)$ for $t \geq T$. We require that: (i) $\mu_Z < 0$, as a necessary and sufficient condition for adoption, i.e., $T < \infty$; and (ii) $\delta - \mu_A > 0$, to have a well defined maximization problem, i.e., expected present value of output is bounded from above after the switch to the new technology has occurred. Notice also that in (2.4), value maximization is over the threshold $b \leq Z$, which also pins down the stopping time $T(b)$. As we will see next, the state variable relevant for the adoption decision is the latent technology Z , and the optimal threshold has the form $(-\infty, b^*]$. We calculate the optimal adoption rule by finding the optimal threshold b^* . To solve (2.4), we first calculate the value before adoption—*inaction value*—and then use two bounday conditions, *value matching* and *smooth pasting*, which hold at the time of adoption and pin down the optimal threshold.

After some manipulations and applying Itô's Lemma, the value for the agent in the inaction region is the following Hamilton-Jacobi-Bellman (HJB) equation (omitting time subscripts):

$$\rho V(Z, A) = e^Z A + \mu_Z V_Z(Z, A) + \frac{1}{2} \sigma_Z^2 V_{ZZ}(Z, A) + \mu_A A V_A(Z, A) + \frac{1}{2} \sigma_A^2 A^2 V_{AA}(Z, A). \quad (2.5)$$

Following Alvarez and Stokey (1998), we exploit the homogeneity of the utility function to rewrite the HJB equation (2.5) in intensive form. Since the utility function is linear in A , $\mathcal{U}(Z, A) = \tilde{\mathcal{U}}(Z)A$, the value function has the same properties such that $V(Z, A) = v(Z)A$. It is now possible to fully characterize the optimization problem of the agent in the space of Z . Using the homogeneity property of the value function, the HJB equation in intensive form is the following,

$$\rho v(Z) = e^Z + \mu v_Z(Z) + \frac{1}{2} \sigma^2 v_{ZZ}(Z), \quad (2.6)$$

where $\rho \equiv \delta - \mu_A > 0$ is the “dividend rate” which, if positive as required for a well-defined maximization problem, generates an opportunity cost of keeping the option to adopt the new technology alive rather than exercise it. The other two parameters of the HJB equation are $\mu \equiv -(2\mu_A + \sigma_A^2) < 0$ and $\sigma^2 \equiv 4\sigma_A^2 > 0$. The HJB equation in intensive form (2.6) is a second-order linear differential equation whose general solution has the form,

$$v(Z) = \frac{e^Z}{\rho - \mu - \frac{1}{2}\sigma^2} + C_1 e^{\xi_1 Z}, \quad (2.7)$$

where C_1 is an unknown constant to be determined, and the parameter ξ_1 is the negative root of the quadratic equation,

$$f(\xi) \equiv \frac{1}{2}\sigma^2\xi^2 + \mu\xi - \rho = 0.$$

The boundary conditions imposed at the time of adoption determine the constant C_1 and the rule for optimally adopting the new technology. The adoption rule is a threshold policy $(-\infty, b^*]$, that is, as long as the latent technology Z is in the inaction region $(b^*, +\infty)$ the new available technology is not adopted. When instead the variable Z crosses the threshold b^* and enters the action region $(-\infty, b^*]$ then the new technology is adopted. The first boundary condition is value matching, which requires that as the state Z approaches the threshold b^* , the value for the agent approaches to the expected present value of future output flows net of the costs of adoption:

$$\lim_{Z \rightarrow b^*} v(Z) = \frac{1}{\delta - \mu_A} - c, \quad (2.8)$$

where $(\delta - \mu_A)^{-1}$ is the expected present value of output immediately after the new technology $A(t)$ is adopted, and $c \in (0, 1)$ is the share of output paid as cost of adoption. After substituting the value of the firm (2.7) into the value matching condition (2.8), we simplify to obtain the boundary condition in terms of the latent technology Z and the threshold b :

$$v(Z, b) = \frac{e^Z}{\rho - \mu - \frac{1}{2}\sigma^2} + \left[\left(\frac{1}{\rho} - c \right) - \frac{e^b}{\rho - \mu - \frac{1}{2}\sigma^2} \right] e^{\xi_1(Z-b)}. \quad (2.9)$$

Finally, the smooth pasting condition requires that

$$\lim_{Z \rightarrow b^*} v_b(Z, b) = 0,$$

that is, we maximize the value (2.9) by choosing the optimal threshold b^* . The maximization of (2.9) with respect to b yields the optimal threshold:

$$b^* = \underbrace{\ln[1 - \rho c]}_{\mathcal{DC}} + \underbrace{\ln\left[1 - \frac{\sigma^2}{J - \mu}\right]}_{\mathcal{SC}} < 0, \quad (2.10)$$

where $\rho \equiv \delta - \mu_A > 0$, $\mu \equiv -(2\mu_A + \sigma_A^2) < 0$, $\sigma^2 \equiv 4\sigma_A^2 > 0$, and $J \equiv (\mu^2 + 2\rho\sigma^2)^{\frac{1}{2}} > 0$. The optimal threshold b^* consists of a deterministic component $\mathcal{DC} = \ln[1 - \rho c]$ and a stochastic component $\mathcal{SC} = \ln[1 - \sigma^2/(J - \mu)]$. The deterministic component \mathcal{DC} is exactly equal to the optimal threshold that one would get in a deterministic model in which $\sigma_A = 0$.

Equation (2.10) is the closed-form solution for the optimal adoption threshold, which allows us to derive the following comparative statics:

$$\frac{\partial b^*}{\partial \mu_A} > 0, \quad \frac{\partial b^*}{\partial \sigma_A} < 0, \quad \frac{\partial b^*}{\partial c} < 0, \quad \frac{\partial b^*}{\partial \delta} < 0. \quad (2.11)$$

The comparative statics is intuitive. On the one hand, as the drift of the income process after adoption μ_A increases, the threshold b^* increases too, leading to earlier adoption. The representative agent trades off the benefits with the costs of adoption. An increase in the drift corresponds to an increase in expected income growth after adoption, leaning then towards earlier adoption. On the other hand, if the volatility σ_A increases, the threshold b^* instead decreases implying later adoption. This latter result is the standard delaying effect of uncertainty at the core of any real options theory of investment. In our context, the adoption decision corresponds to exercising a call option, whose value increases with the volatility of the underlying asset, which is the frontier technology in our case. From equation (2.10), it is clear that uncertainty

about the future—as summarized by the parameter $\sigma = 2\sigma_A$ —effectively acts as a *barrier* to technology adoption.

Finally, as the adoption cost c and the time discount factor δ increases, the threshold b^* moves leftward implying later adoption. Clearly, if the fraction c of forgone output paid as adoption cost increases then the agent waits longer to adopt the new technology. With a higher time discount factor, the agent attaches relatively more weight to the present, such that she prefers to postpone the payment of the cost and so adoption.

The solution of the model can also be written in terms of expected time needed for the new technology to be adopted. Given the threshold (2.10) and the stochastic process for the latent technology Z in (2.3), the expected optimal stopping time is

$$\mathbb{E} [T(b^*)] = \frac{b^*}{\mu_Z}, \quad (2.12)$$

where $\mu_Z = \frac{1}{2}\sigma_A^2 - \mu_A < 0$, and we normalize the initial condition to $Z_0 = 0$. The optimal stopping time rule (2.12) tells how much time it takes for the new technology to be adopted—*time adoption lag*.

To summarize, this section shows that the notion of time adoption lags naturally raises from a simple model of technology adoption. The model provides a closed-form solution for the time adoption lag as a function of adoption costs, discount rate, drift and volatility of the income process after adoption.

2.2.3 Numerical Comparative Statics

In this section, we conduct a numerical comparative statics exercise to understand the relative importance of the various forces at work in the model. We assign numerical values to the parameters of the model and analyse the sensitivity of the model-implied time adoption lag in (2.12) to a specific parameter. Table 3.1 contains the

baseline parameter values.

Recall that in the model the drift and volatility (μ_A, σ_A) of the stochastic process for the technology in use correspond to the drift and volatility of output. Specifically, the values $\mu_A = 1.6\%$ and $\sigma_A = 4.4\%$ are chosen based on our estimates of respectively mean and standard deviation of annual growth rates of real GDP per capita in a sample of 60 countries for 1973-2002 [see Table 2.2, column (1)]. A value of $\delta = 5\%$ corresponds to a 5% annual interest rate. We set the adoption cost $c \in (0, 1)$ to 0.5 of output at the time of adoption. We do not have any reference value guiding our choice of the adoption cost so we set it at the middle of the possible range.

Figure 2.1 contains the results of the numerical comparative statics. In Panel A, the model predicts a strong positive relationship between volatility and time adoption lags. This positive relationship speaks of a large real value of inaction. Notice how sensitive is the time adoption lag to variations in volatility: going from a standard deviation of 4% to 8% per year leads to a more than three times larger time adoption lag, specifically it goes from approximately 7 to 26 years. Importantly, such a variation in volatilities—4% to 8%—is only a conservative range if compared to cross-country data. For instance, in a sample of 60 developing and developed countries [Table 2.2, column (1)], the least and most volatile countries have respectively a standard deviation of annual growth rates of real GDP per capita of 1.4% and 12.1%.

Taken at face values, the comparative statics results suggest that uncertainty about the future income process has quantitatively large negative effects on the adoption of a new technology. Moreover, volatility is by far the most relevant margin for the adoption decision. We acknowledge that, at first glance, the adoption lags implied by the model seem enormous. However, our own empirical evidence [see Tables B.1, B.2 and B.3] and many authors have documented adoption lags of this order of magnitude. For instance, Jovanovic and Rousseau (2005) report that the process of Electrification took roughly 35 years, and the IT revolution started in the

early 70's and still now pervades modern economies.⁷

Panel B portrays a negative relationship between the drift of the income process after adoption and the time adoption lag. As the drift increases, future income growth outweighs the adoption cost and the uncertainty about future income such that the time adoption lag decreases. This latter result implies a negative relationship between mean growth and time adoption lags.

Notice that volatility has not only an adverse effect on the diffusion of the new technology but, as a consequence, also a clear negative effect on growth. By delaying the adoption of the new technology, volatility adversely affects the growth opportunities generated by the arrival of the new technology. This negative effect would be identified in the data by exploiting the cross-country variation in volatility and mean growth, which is exactly the estimation strategy put forward by Ramey and Ramey (1995).

Finally, Panel C and D depict the relationship between time adoption lag, adoption cost and time discount factor, respectively. In Panel C, as the adoption cost increases the incentives for early adoption are curtailed, such that the time adoption lag increases accordingly. In Panel D, a positive relationship also holds for the time adoption lag and the time discount rate δ . If the representative agent cares relatively more about the present than the future—higher δ —then she has incentives to postpone the payment of the cost and adopt the new technology later in time. Interestingly, the model predicts that both adoption costs and discount rates are quantitatively far less important than volatility.

⁷ Rosenberg (1976), Devine (1983), David (1990), and Atkeson and Kehoe (2007) report similar numbers.

2.2.4 Empirical Predictions

In this section, we detail the empirical predictions of the model. Later in Section 2.4 and 2.5 of the paper, we present empirical evidence for a large sample of developing and developed countries which speaks to each of the following implications of the model.

Prediction 1. *There is a positive relationship between time adoption lags and volatility.* In the model, the interaction of uncertainty about the future income process with irreversibilities in technology adoption investment generates a real value of inaction which delays the switch to the new technology. Hence, we would expect countries with more volatile growth rates of real GDP per capita to have higher time adoption lags. As Section 2.5 below shows, this is indeed the case in the data.

Prediction 2. *There is a negative relationship between volatility and growth.* In the model under our simplifying assumptions, there is no output growth before adoption of the new technology. Instead, the new frontier technology guarantees a positive trend growth. This implies as volatility delays adoption of the new technology, it also adversely affects the average growth rate of output.⁸ Hence, we would expect countries with more volatile growth rates of real GDP per capita to have lower average growth rates of real GDP per capita. As Section 2.5 below shows, this is indeed the case in the data.

Prediction 3. *There is a negative relationship between time adoption lags and growth.* In the model, an increase in the drift of the income process after adoption leans towards early adoption. Hence, we would expect countries with higher growth rates of real GDP per capita to have lower time adoption lags. As Section 2.5 below shows, this is indeed the case in the data.

⁸ This conclusion still holds if we allow for a positive drift of the old technology as long as the new frontier technology dominates the old one in use.

The three predictions combined convey an important message. In a model in which technology adoption is the decision of forward-looking agents, technology diffusion, growth and volatility are jointly determined as an equilibrium outcome. The relevant driving force are agents' expectations about the benefits they can reap after new technology is adopted.

The model also has stark predictions on the dynamics of the cross-country dispersion of income levels.

Prediction 4. *After a new technology is introduced, international income levels start to diverge until the diffusion process is completed.* Suppose we index the time of adoption T by country, i.e., $\{T_i\}_{i=1}^I$. We envision a world economy composed by I independent economies. Furthermore, suppose countries differ by their structural parameters $(\mu_A^i, \sigma_A^i, c_i, \delta_i)_{i=1}^I$. As a new technology is introduced, parameter heterogeneity leads to cross-country dispersion in time adoption lags and so dispersion in output levels. Hence, we would expect the cross-country dispersion in incomes per capita to increase at the onset of a technological revolution. As Section 2.4 below shows, this is indeed the case of personal computers.

2.3 Data Description

This section provides the description of the variables and the sources of all the data used in the following empirical analysis. Our measure of country-specific volatility is the standard deviation of per capita real GDP annual growth rates. We define the real growth rate as $g_{it} = \Delta \ln(y_{it})$, where y_{it} is the Purchasing Power Parity (PPP) converted GDP per capita measured at 2005 constant prices. For each country i in our sample, we calculate the mean and the standard deviation of the real growth rate as $\bar{g}_i = \frac{1}{T} \sum_{t=1}^T g_{it}$ and $\sigma_i = \left[\frac{1}{T} \sum_{t=1}^T (g_{it} - \bar{g}_i)^2 \right]^{1/2}$. Data on real GDP per capita are from the Penn World Tables (PWT) Version 7.0. Regarding the diffusion of

major technologies, we focus on three Information and Communication Technologies (ICT's). The information technologies (IT's) considered are *personal computers* and *internet*, the communication technology (CT) is *cell phones*. All technology data are from the Cross-country Historical Adoption of Technology (CHAT) dataset whose detailed description can be found in Comin and Hobijn (2009).

The exact definition of the technology variables is the following: 1) *personal computers* is the number of self-contained computers designed for use by one person. Invention date: 1973; 2) *internet* is the number of people with access to the worldwide web. Invention date: 1983; and 3) *cell phones* is the number of portable cell phones. Invention date: 1973. We measure the intensity of usage of these three technologies in per capita terms. Data on population counts are from PWT Version 7.0.

The three ICT's we consider in this paper represent an ideal starting point for an empirical study of technology diffusion for at least three reasons. First, they have been introduced recently such that we can match data on their diffusion with reliable GDP data from PWT. Second, computers and cell phones are a clear case of embodied technology: a country can not adopt the computing and communication power embodied in these technologies without physically installing computers or buying cell phones. The internet is a technology closely related to the adoption of computers such that we regard the number of internet users as another measure of diffusion of embodied technical change. Third, computers and internet are General Purpose Technologies (GPT's).

Since the seminal work of Griliches (1957), one of the main obstacles to the study of technology diffusion has been data availability. Most of the applied microeconomic literature on technology diffusion focused on the estimation of reduced-form diffusion curves for a relatively small number of technologies and countries. Moreover for data availability reasons, the focus of this early literature has been exclusively on the extensive margin of technology adoption. The CHAT dataset allows us to expand

the set of countries included in the sample and to consider both the extensive and intensive margin of technology adoption. Unfortunately given the focus of this work and despite the availability of better refined data, for most countries the time series dimension of the three ICT's is still too short to conduct a full-blown time series empirical exercise. For the latter reason, we will mainly rely on the cross-sectional variation of the data.

The main dependent variable of our cross-sectional exercise is a measure of time adoption lags proposed by Comin et al. (2008). This measure has two clear advantages: 1) time lags are independent of the units of measurement; and 2) the calculations of these lags only require a long time series for the country leader in a specific technology. For two of our technologies, computers and internet users per capita, the U.S. is the country leader. For cell phones per capita, even though the U.S. is not the leader, we keep it as the country benchmark in order to consistently compare time usage lags across technologies. We believe that measuring technology diffusion lags in years is quite intuitive and gives a clear picture of what being a laggard country means.

Time Adoption Lags. Let k_{it^*} be the per capita usage level of a given technology in country i at the benchmark year t^* . We compare the observation k_{it^*} with the time series for the U.S., $k_{US,s}$ where s indexes the observations. Let S denote the set of observations available in the time series for the U.S. Then, we define the following two observations in the U.S. time series. The first observation \bar{s} is the last time the U.S. passed usage level k_{it^*} :

$$\bar{s} = \arg \min_{s \in S} \{s | k_{US,s'} \geq k_{it^*}\}$$

for all $s' \in S$ and $s' \geq s$. The second observation \underline{s} is the last time the U.S. recorded a technology usage level lower than or equal to k_{it^*} :

$$\underline{s} = \arg \max_{s \in S} \{s | k_{US,s'} \leq k_{it^*}\}.$$

We impute the time that the U.S. last had the technology usage level k_{it^*} , say τ , by linear interpolation:

$$\tau = \left(\frac{k_{it^*} - k_{US,\underline{s}}}{k_{US,\bar{s}} - k_{US,\underline{s}}} \right) \bar{s} + \left(\frac{k_{US,\bar{s}} - k_{it^*}}{k_{US,\bar{s}} - k_{US,\underline{s}}} \right) \underline{s}.$$

The technology usage lag between the U.S. and country i at the benchmark year is then given by $Lag_i \equiv t^* - \tau$. Specifically, we use $t^* = 2002$ as the benchmark year for all three technologies. Hence, the variable Lag_i measures how many years ago the U.S. last had the usage technology level that country i had in 2002. Finally, one last note on the method used to construct time adoption lags is needed. The method described above exploits only the time series dimension of the U.S. series arising then the possibility of right-truncated adoption lags: for some countries, the technology usage per capita levels in the benchmark year 2002 are lower than any available observation for the U.S.⁹ These countries are those that operate way inside the technology frontier. For these countries time adoption lags cannot be calculated. However, we know that their adoption lags are larger than the maximum adoption lag we observe in our sample (i.e. right-truncated lags) and that these countries are relatively poor countries in terms of GDP per capita. We do not take any action to formally handle right-truncation and we restrict our sample to countries for which we can calculate adoption lags. For cell phones per capita instead, time adoption lags are censored because the U.S. never reaches the usage level that some countries have in the benchmark year 2002. This happens because the U.S. is not the technological leader for cell phones.

⁹ The first available observation for the U.S. is dated 1981 for personal computers, 1984 for internet users and 1984 for cell phones.

A criticism to the time adoption lags described above, particularly relevant for the focus of the paper, is that their calculation hinges entirely on the usage level of a specific technology without any adjustment for the quality improvements that technologies experience over time. For example, a personal computer bought in the 2002 has much more computing power than one installed in the early eighties. Since the focus of the paper is also on cross-country differences in productivity, we calculate quality-adjusted time adoption lags. Furthermore, if relatively poorer countries are catching up with the technological leader, they could be installing better quality machines (i.e. newer vintages) such that unadjusted time adoption lags would over-estimate the actual distance from the frontier. Among the three technologies we consider, computers and cell phones are embodied technologies such that it would be sensible to calculate their adjusted adoption lags. However, due to data availability, we are able to calculate quality-adjusted time adoption lags only for personal computers.

Quality-Adjusted Time Adoption Lags. The only difference from the calculations described before is the benchmark per capita usage level k_{it^*} . For instance in the computers case, the unadjusted usage level k_{it^*} was simply the number of personal computers per capita. Let $k_{it^*}^{adj}$ denote the quality-adjusted stock of computers per capita. For each country i , we calculate $k_{it^*}^{adj}$ as,

$$k_{it^*}^{adj} = k_{it_0} + \sum_{j=1}^{t^*-t_0} \gamma^j I_{t_0+j},$$

where $I_{t_0+j} = k_{t_0+j} - (1-\delta)k_{t_0+j-1}$, γ is the (gross) growth rate of embodied technical change, $t^* = 2002$ and $t_0 = 1981$ are respectively the benchmark year and the date of the first available U.S. observation for personal computers. We have normalized the t_0 's quality level at 1. The growth rate of the technological frontier is calibrated

to the mean growth rate of the U.S. *investment-specific* technical change over the period 1973-2002, that is, $\gamma = (1 + 9.15\%)$.¹⁰ We are then taking the U.S. as the technological frontier and assuming that the best vintage is freely available to all countries. Hence, cross-country differences in quality-adjusted stocks of machines emerge because of cross-country differences in diffusion lags. We set the depreciation rate $\delta = 12.4\%$.¹¹

In calculating the quality-adjusted time adoption lag, we make the following assumptions: (i) all new machines, I_{t_0+j} , are of the latest vintage, that is, they are frontier technology-embodying capital goods; (ii) for each country, personal computers of the first available observation are all frontier machines, that is, computers at time $t_0 + j \geq t_0$ have quality γ^j , no matter whether $t_0 + j$ is the date of the first available observation for the country. Assumption (i) is obviously wrong if firms invest in vintages older than the frontier vintage. However, this would create problems only if there are cross-country differences in firms investment behavior.¹² If relatively poor countries tend to purchase older vintages more frequently than rich countries then our quality-adjusted time adoption lags would underestimate their actual distance from the frontier (i.e. the U.S.) creating then issues in the cross-country comparison of the time adoption lags. If on the other hand, poor countries buy older machines less often than rich countries, circumstance we think unlikely, then we would overestimate their distance from the technological frontier. Assumption (ii), instead, tends to underestimate the technological gap of laggard countries. This happens because

¹⁰ Following Greenwood et al. (1997), we calculate investment specific technical change as $q_t = p_t^c/p_t^i$, where p_t^c is the implicit price deflator for nondurable consumption goods and p_t^i is the price index for computers and peripheral equipment. Both price series are from the Bureau of Economic Analysis (BEA). As reported in Greenwood et al. (1997) and Cummins and Violante (2002), the BEA provides one of the most reliable constant-quality price indexes for personal computers.

¹¹ Greenwood et al. (1997) obtain a physical depreciation rate of $\delta = 12.4\%$ for equipment. They use capital stock data from BEA.

¹² The empirical evidence on this regard is scarce. Goolsbee (1998) documents that some sub-Saharan African countries purchased old vintage airplanes during the period 1988-1993.

laggard countries are also those whose time series starts later in time.

For the cross-country regressions of Section 2.5, we have three separate samples: 1) 60-country sample from 1973 to 2002 (Computers Sample); 2) 61-country sample from 1983 to 2002 (Internet Sample); and 3) 68-country sample from 1973 to 2002 (Cell Phones Sample). The three data samples do not necessarily contain the same countries and their respective sample periods start from the invention year of the specific technology under consideration and end in the benchmark year 2002. More precisely, to build these samples we follow two steps. First, we start from a panel of countries that has a complete time series for the real GDP per capita since 1960. According to this criterion the PWT produces a balanced sample of 110 countries over the time span 1960-2009. Second, we construct time adoption lags starting from the derived sample of 110 countries. The smaller number of countries present in the three final samples is entirely due to the impossibility to calculate adoption lags for some of the countries. Tables B.1, B.2 and B.3 in Appendix B.1 contain respectively computers, internet and cell phones time adoption lags for every country in each of the three data samples.

When we use quality-adjusted time adoption lags, the Computer Sample contains a larger number of countries, 83 instead of 60. This happens because quality adjustment alleviates the right-truncation problem we discussed previously in this section.

2.4 Technology Diffusion and International Incomes

In this section, we present evidence on the link between technology diffusion and international incomes per capita. The aim of this section is to demonstrate that time adoption lags are a sensible measure of long-run economic performance. Figure 2.2 shows a strikingly strong cross-country negative relationship between personal

computers adoption lags (x-axis) and 2002 real GDP per capita (y-axis). Relatively poor countries are also laggards in the diffusion process of personal computers.

Figure 2.3 shows that the distributions of incomes per capita and computers time adoption lags are twin-peaked. While bimodality is now a well-known property of the cross-country income distribution, to the best of our knowledge, polarization in the distribution of diffusion lags is a new fact.

However, what is relevant for the aim of this section are the properties of the joint density of international incomes per capita and time adoption lags. Importantly, Figure 2.4 shows that also the joint distribution is bimodal. Hence, there is not only a linear negative relationship between standard of living and diffusion lags (see Figure 2.2), but countries are polarized in two clubs also in the income-diffusion dimension: rich countries closer to the technological frontier (lower time adoption lags) versus poor countries lagging farther behind (higher time adoption lags).

Figures 2.2, 2.3 and 2.4 are a static picture of the relationship between diffusion lags and cross-country differences in incomes per capita. However, cross-country differences in technology diffusion should also affect the dynamics of the cross-country dispersion in incomes per capita. Figure 2.5 tracks the cross-country dispersion of international incomes per capita over time.

Figure 2.5 plots the cross-country standard deviation of the logarithm of real GDP per capita (red solid line) and real GDP per worker (labor productivity, blue dashed line) over time. Specifically, we calculate the cross-country mean as $\mu_t = \frac{1}{I} \sum_{i=1}^I \ln(y_{it})$, where y_{it} is the PPP converted GDP per capita and GDP per worker, measured at 2005 constant prices. $I=60$ is the total number of countries in the computer sample. We then calculate the cross-sectional standard deviation as $\sigma_t^{\text{cs}} = \left[\frac{1}{I} \sum_{i=1}^I (\ln(y_{it}) - \mu_t)^2 \right]^{1/2}$, for $t = 1960, \dots, 2002$.

The dynamics of the cross-sectional standard deviations in Figure 2.5 show in-

interesting patterns. For the real GDP per capita (red solid line), the cross-sectional standard deviation has been fairly stable from the 1960 to the early 80's and starts to sharply increase since the mid 80's. A similar pattern also emerges for the real GDP per worker (blue dashed line). The cross-sectional standard deviation of labor productivity is fairly stable from the 1960 to late 70's, it then steadily decreases (i.e., *sigma-convergence*) reaching the trough in 1981, and finally it starts to sharply increase since 1981 to the end of the sample. The dynamics shown in Figure 2.5 are particularly interesting since the turning point, that is the year 1981, is in the middle of the IT revolution, and arguably the beginning of the diffusion process of personal computers.¹³ For most countries in our sample, the diffusion of computers has been extremely limited between the 70's and 90's. We conjecture that the increasing dispersion in incomes per capita and labor productivity across countries is the result of the cross-country differences in the speed of diffusion of the IT revolution.

Major technological innovations, such as the IT revolution, act like shocks to the cross-country distribution of incomes per capita.

2.5 Evidence on Volatility and Technology Adoption

This section provides cross-country and time series evidence on the link between volatility and technology diffusion. As in Ramey and Ramey (1995), we use the standard deviation of output growth as our measure of volatility. We acknowledge that other volatility measures can be considered. Volatility in the national stock markets and measures of macro policy variability are all good candidates. Stock market volatility has been used by Bloom (2009) to capture macroeconomic uncertainty. In a context of cross-country growth regressions, Kormendi and Meguire (1985) and Grier and Tullock (1989) use respectively the standard deviation of money supply

¹³ Interestingly, the first available observation for computers is for the U.S. and it is dated 1981.

shocks and inflation as a proxy for monetary policy variability. Ramey and Ramey (1995) derive a measure of fiscal policy variability from a government spending forecasting equation. Tabova and Burnside (2010) look at the exposure to global factors as a source of country-specific risk. However, all these different sources of volatility eventually affect output, such that our analysis can be interpreted as assessing the combined effect that all these sources of volatility have on technology adoption.

Cross-country differences in technology adoption are measured by time adoption lags in the diffusion of three major ICT's: personal computers, internet and cell phones.

2.5.1 Cross-Country Evidence

This section presents the empirical strategy and discusses the findings from the cross-sectional regressions. The estimating equation is the following:

$$Lag_i^j = \alpha_j + \beta_j Growth_i + \gamma_j Volatility_i + \epsilon_i^j. \quad (2.13)$$

We estimate equation (2.13) by Ordinary Least Squares (OLS) separately for each technology $j \in \{computers, internet, cell\ phones\}$. The dependent variable Lag_i^j denotes the time adoption lag of country i for technology j . It measures how many years ago the U.S. last had the usage level of technology j that country i had in 2002. The variables $Growth_i$ and $Volatility_i$ are respectively country i mean, g_i , and standard deviation, σ_i , of per capita real GDP growth rates over the sample period from the invention date of a specific technology and the benchmark year 2002. ϵ_i^j is a country i , technology j unexplained error term. Table 2.2 contains descriptive statistics for the three variables included in equation (2.13).

Even though we are interested in the coefficient on *Volatility*, we control for cross-country differences in mean growth for two reasons: first, we would like to interpret our results in the spirit of a mean-preserving spread across countries. As shown

by Ramey and Ramey (1995), there is a robust negative cross-country relationship between the mean and standard deviation of output growth in a 92-country sample and over the period 1962-1985. By controlling for *Growth* in equation (2.13), we estimate the coefficient of interest γ_j netting out the direct effect that volatility has on growth. To check whether in our three samples there is the same statistically significant relationship between volatility and growth, we re-estimate the volatility-growth regressions of Ramey and Ramey (1995). Table 2.3 confirms the previous findings of a negative relationship between volatility and growth and in fact justifies our specification.

Second, by having the variable *Growth* on the right hand side of equation (2.13), we are implicitly controlling for all the covariates that have been shown to matter in cross-country growth regressions. This is a very appealing feature of the estimating equation (2.13) since the literature on cross-country growth regressions is vast, and so the number of relevant covariates is extremely large.¹⁴

Tables 2.4, 2.5 and 2.6 show our main empirical findings. Table 2.4 shows the parameter estimates of equation (2.13) for personal computers per capita. Columns (1)-(3) are for the full Computer Sample of 60 countries, columns (4)-(6) are for a subsample of 24 OECD countries. We focus our discussion on our favorite specification, that is, columns (3) and (6). The coefficient on *Volatility* has a positive sign in both samples and it is highly statistically significant with a probability value of zero. More volatile countries have higher time adoption lags in computers per capita, that is, they are relatively farther behind the technological frontier. Importantly, the positive sign persists even after controlling for cross-country differences in mean growth, suggesting that volatility has a direct adverse effect on technology diffusion independently of the adverse effect it has through diminished growth. This

¹⁴ Levine and Renelt (1992) implement an extensive sensitivity analysis of the cross-country growth regressions.

latter observation is important since it highlights a potential *level effect* as opposed to a *growth effect* of volatility, which to the best of our knowledge has not been documented by the literature. By *level effect*, we mean the adverse effect that volatility has on a country's TFP through the delaying effect it has on the adoption of better technologies.

The coefficient on *Volatility*, $\gamma_{computers} = 137.222$ implies that a 1 percentage point increase in volatility is associated to an increase in the time adoption lag of approximately 1.37 years.

By looking at Table 2.2 [Panel C, column (1)], we see that in the Computer Sample the difference between the volatility of the highest (Nicaragua) and lowest-*Volatility* country (France) is approximately 8 percentages points. This difference in volatility implies a difference in adoption lags of almost 11 years. A difference of 11 years in time lags between the highest and lowest volatility country, net of growth effects, seems considerable in magnitude and points toward a potentially quantitatively important mechanism to investigate. The high statistical significance of the volatility coefficient also holds in the OECD sample. Moreover, the magnitude of the estimated coefficient is even larger than that estimated in the full sample of 60 countries. The estimates suggest that in the OECD sample a 1 percentage point increase in volatility is associated with an increase in the lag of technology adoption of approximately 3.25 years. The maintained positive sign, statistical significance and an even larger magnitude of the volatility coefficient suggests that the positive link between volatility and time adoption lags persist within a relatively more homogeneous set of countries. This latter finding is comforting since the regressions in the OECD sample can be interpreted as controlling for an OECD-fixed effect.

The positive relationship between volatility and time adoption lags is found also for the other two technologies we consider. Tables 2.5 and 2.6 show the results for respectively internet and cell phones per capita. Since a negative statistically

significant relationship between volatility and growth is found also in the Internet and Cell Phones Sample [see Table 2.3, columns (2)-(3)], as before we only discuss the results of the specification with both *Growth* and *Volatility* as explanatory variables. The volatility coefficient, $\gamma_{internet}$ is statistically significant at the 1 percent level in the full sample of 61 countries, and at the 5 percent level in the OECD sample of 23 countries. As in the computers per capita case, the magnitude of the volatility coefficient in the OECD sample is larger than that in the full sample.

Finally, Table 2.6 shows that similar results hold for cell phones per capita. In this latter case, we only show results for the full sample of 68 countries. As already discussed in Section 2.3, because of censoring, the Cell Phones Sample contains only 4 OECD countries. This happens because the U.S. is not the country leader in this specific technology. Given the OECD results for computers and internet users per capita, the lack of OECD countries in the Cell Phones sample could also explain the weaker statistical significance and smaller magnitude of the coefficient on volatility. However, the coefficient remains positive and still significant at the 10 percent level.

Past Volatility and Time Adoption Lags. As in any reduced-form estimation like that of equation (2.13), any attempt to make causal inference is irremediably flawed because of endogeneity issues. Conscious of this issue, we re-assess the empirical link between volatility and technology adoption by calculating a measure of past growth and volatility performance. The only difference with the estimating equation (2.13) is that now the explanatory variables *Growth* and *Volatility* are calculated over the period prior to the invention of the specific technology. More precisely, for each country i , we calculate the mean and the standard deviation of the real GDP growth rates as $g_i = \frac{1}{T^j - 1960} \sum_{t=1960}^{T^j} g_{it}$ and $\sigma_i = \left[\frac{1}{T^j - 1960} \sum_{t=1960}^{T^j} (g_{it} - g_i)^2 \right]^{1/2}$, where T^j is the invention year of technology $j \in \{computers, internet, cell\ phones\}$. As forcefully shown by Table 2.7, the coefficient on volatility is statistically significant at

the 1 percent level for all three technologies. The magnitudes of the coefficients are comparable to those estimated and discussed previously in this section. Remarkably, these latter results confirm and actually strengthen the case of a positive relationship between volatility and time lags in the diffusion of new technologies.

Volatility and Quality-Adjusted Time Adoption Lags. The empirical findings we presented so far are all based on a measure of time diffusion lags that neglects improvements in the quality of the new available machines. To address this potential drawback, we re-estimate equation (2.13) with quality-adjusted time adoption lags (QA-Lag's) as dependent variable:

$$QA - Lag_i = \alpha + \beta Growth_i + \gamma Volatility_i + \epsilon_i. \quad (2.14)$$

The superscript j is omitted since we estimate (2.14) only for personal computers. The estimates in Table 2.8 confirm the previous findings of a positive and highly statistically significant relationship between volatility and time adoption lags. The volatility coefficient γ has a probability value of zero in both the full sample of 83 countries, column (3), and the OECD sample of 24 countries, column (6). The magnitude of the volatility coefficients, 94.686 and 218.498 respectively in the full and OECD sample, is smaller than that estimated with the un-adjusted time adoption lags [Table 2.4, column (3) and (6)]. This is consistent with our criticism of the un-adjusted time adoption lags over-estimating the actual distance from the frontier technology. Importantly, the negative relationship between volatility and technology diffusion remains highly statistically and economically significant also with our new measure of quality-adjusted time adoption lags.

Robustness. We now briefly discuss two additional sets of results that further corroborate the robustness of our empirical findings. So far mean and standard deviation of real GDP growth rates have been calculated over the period that starts

from the invention year of each specific technology to the benchmark year 2002. Now instead, we use an extended time horizon that starts from the first available observation in our dataset, that is 1960. Besides this small change, the estimating equation (2.13) is left unchanged. Tables B.4, B.5 and B.6 in the Appendix B.1 report the results for all three technologies and, when possible, separate estimates for the full and the OECD sample. Maybe not surprisingly, this first robustness check confirms our previous findings.

The second robustness check consists in changing, from 2002 to 1995, the benchmark year for the calculation of the time adoption lags. One important shortcoming in changing the benchmark year to 1995 is the diminished number of observations. By switching the benchmark year far back in the past, we aggravate the right-truncation problem in adoption lags that we previously discussed in Section 2.3: we are unable to calculate time adoption lags for those countries whose diffusion process occurred largely after the new benchmark year 1995. Specifically, the sample that suffers the most from the change in the benchmark year is the Internet Sample that after the change only contains 19 countries. The other two data samples, Computers and Cell Phones Sample, lose respectively 20 and 21 observations relative to the 2002 benchmark case. Table B.7, in the Appendix ?? at the end of the paper, contains the results of the second robustness check. For computers and cell phones per capita, respectively column (1) and (3), the coefficients on volatility are statistically significant at the 1 percent level and with the right positive sign. For internet per capita, column (2), instead none of the coefficients is statistically significant even though the coefficient on volatility still maintains the positive sign. We conjecture that a potential explanation for the lack of statistical significance in the Internet sample could be its reduced size.

To summarize, we have shown that: (i) there is a highly statistically significant negative relationship between volatility and technology diffusion—volatile countries

are also laggards in the adoption of new technologies; (ii) the positive relationship between volatility and time adoption lags holds for each of the three major information and communications technologies we consider; (iii) countries that have been relatively more volatile prior to the invention of a specific technology are also the ones farther behind in the diffusion process of that specific technology; and (iv) the empirical findings are quite robust to changes in the benchmark year of the time adoption lags, and in the sample period over which mean growth and volatility are calculated.

2.5.2 Time Series Evidence

This section presents the empirical findings from panel regressions. In Section 2.5.1 above, the identification of the link between volatility and technology diffusion is entirely based upon the cross-country variation in volatility, growth rates and diffusion lags. In this section instead, we exploit the time series dimension of the data after netting out country-fixed effects.

The dependent variable is no longer the time adoption lags of Section 2.5.1, but simply the growth rate in the usage level of a specific technology. Let I_i^j denote the usage level of technology $j \in \{\text{computers}, \text{internet}, \text{cell phones}\}$ in country i . We define the growth rate of technology usage as $I_{it}^j = \Delta \ln(k_{it}^j)$, where k_{it}^j is the usage level in per capita terms of technology j in country i at time t . Since computers and cell phones are two examples of technologies embodied in capital goods, we measure their usage level as the number of machines per capita embodying those technologies, that is, number of personal computers and cell phones per capita. For the internet, instead, we measure its usage level as the ratio of the number of users to population.

For almost all the countries in our samples, the diffusion of all three technologies is extremely limited between the invention year and early nineties. Hence, we calculate the growth rates of technology usage, our dependent variable, over the shorter sample

period 1995-2002. One obvious shortcoming of this choice is that the time-series dimension is too short to provide reliable parameter estimates, and consequently largely limiting the scope of the panel regression analysis. Regarding the explanatory variables, we calculate time varying measures of mean growth and volatility. For each country i , we compute time varying means and standard deviations of the real per capita GDP growth rate as: (i) 5-year rolling (asymmetric) moving averages of the real growth rate as $g_{it}^{roll} = \frac{1}{5} \sum_{j=0}^4 g_{it-j}$ for $t = 1995, \dots, 2002$, and (ii) $\sigma_{it}^{roll} = \left[\frac{1}{5} \sum_{j=0}^4 (g_{it} - g_{it}^{roll})^2 \right]^{1/2}$ for the same sample period $t = 1995, \dots, 2002$. Among the right-hand-side variables we also include a set of year dummies D_t .

The estimating equation is the following:

$$I_{it}^j = \alpha_j + \beta_j g_{it}^{roll} + \gamma_j \sigma_{it}^{roll} + \delta_j D_t + \epsilon_{it}^j. \quad (2.15)$$

We estimate equation (2.15) by Fixed Effects (FE) separately for each technology j . The error term $\epsilon_{it}^j = \eta_i + u_{it}$ consists of a country fixed-effect η_i and a disturbance u_{it} independently and identically distributed among countries and years.

Table 2.9 shows the results from the FE model. In the computers case, $\gamma_{computers}$ is negative and statistically significant only at the 10 percent level in the full sample of 60 countries, column (1), but not in the OECD sample of 24 countries, column (2). After controlling for country-fixed effects, periods of relatively high volatility are associated with periods of lower growth rates in the usage level of computers per capita. The coefficient on time-varying volatility is negative but not statistically significant also in the internet OECD sample, column (5). For the internet full sample and cell phones, the coefficient on our rolling measure of volatility is positive even though never statistically significant. The evidence from panel data regressions is not clear-cut. We propose two reasons for the weak evidence in the time series: 1) the time-series dimension of the data is far too short to provide reliable parameter

estimates; and 2) the country dummies in the FE model absorb most of the variation in the data, making relatively hard the identification of the parameters of interest.

Previous empirical studies have already documented that the relationship between volatility and economic growth is not as strong in the time series as in the cross-section of countries. To this regard, our results using direct measures of technology diffusion confirm this finding.

2.6 Conclusions

I present novel cross-country evidence on the link between volatility and the diffusion of three major information and communication technologies (ICT's)—personal computers, internet and cell phones. First, we show there exists a strong negative relationship between cross-country diffusion lags and international incomes per capita. Maybe not surprisingly, poor countries in the income dimension are producing inside the technological frontier. Furthermore, the joint distribution of international incomes per capita and diffusion lags is twin-peaked. While after Danny Quah's influential work the "twin-peakedness" of the cross-country distribution of incomes per capita is now well-known, the twin-peakedness of the joint distribution of international incomes and diffusion lags is a new fact. This finding is important because demonstrates that the time adoption lags of the three ICT's we consider in this paper are a sensible measure of long-run economic performance. After establishing the existence of a close link between the technologies we consider and the world income distribution, we document a highly statistically and economically significant cross-country negative relationship between volatility and technology diffusion. More volatile countries are farther behind the technological frontier, the U.S. in our case. The empirical findings corroborate the negative relationship between volatility and economic growth first documented by Ramey and Ramey (1995). Moreover, by focusing on direct measures of technology, we highlight a potential mechanism through

which business cycle volatility adversely affects the long-run economic performance of a country. We also offer a simple real options model of technology adoption that is qualitatively consistent with the facts documented. In the model, the interaction of uncertainty and sunk costs of adoption creates a real value of inaction which delays the adoption of new technologies with the consequent adverse effect on long-run economic growth.

2.7 Tables and Figures

This section contains the main tables and figures of the essay.

Table 2.1: Baseline Parameters

σ_A	μ_A	c	δ
4.4%	1.6%	0.5	5%

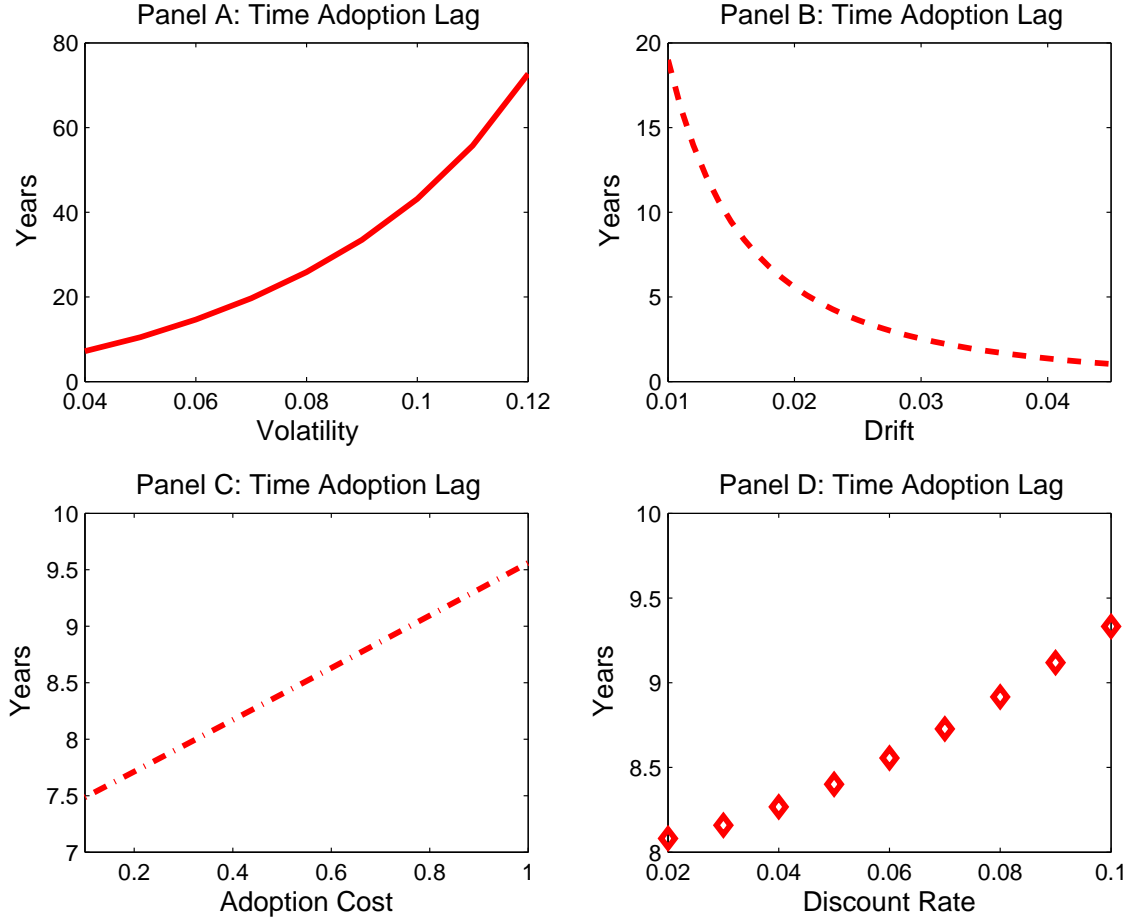


FIGURE 2.1: Numerical Comparative Statics

Notes: Panel A, B, C and D differ only for the comparative statics parameter on the x-axis. The calibration is at an annual frequency: time discount rate $\delta = 5\%$, adoption cost as share of output $c = 0.5$, drift and volatility of the geometric Brownian motion for the income process are respectively $\mu_A = 1.6\%$ and $\sigma_A = 4.4\%$. *Time Adoption Lag* is the optimal stopping time (2.12) implied by the stochastic model of technology adoption in Section 3.2.

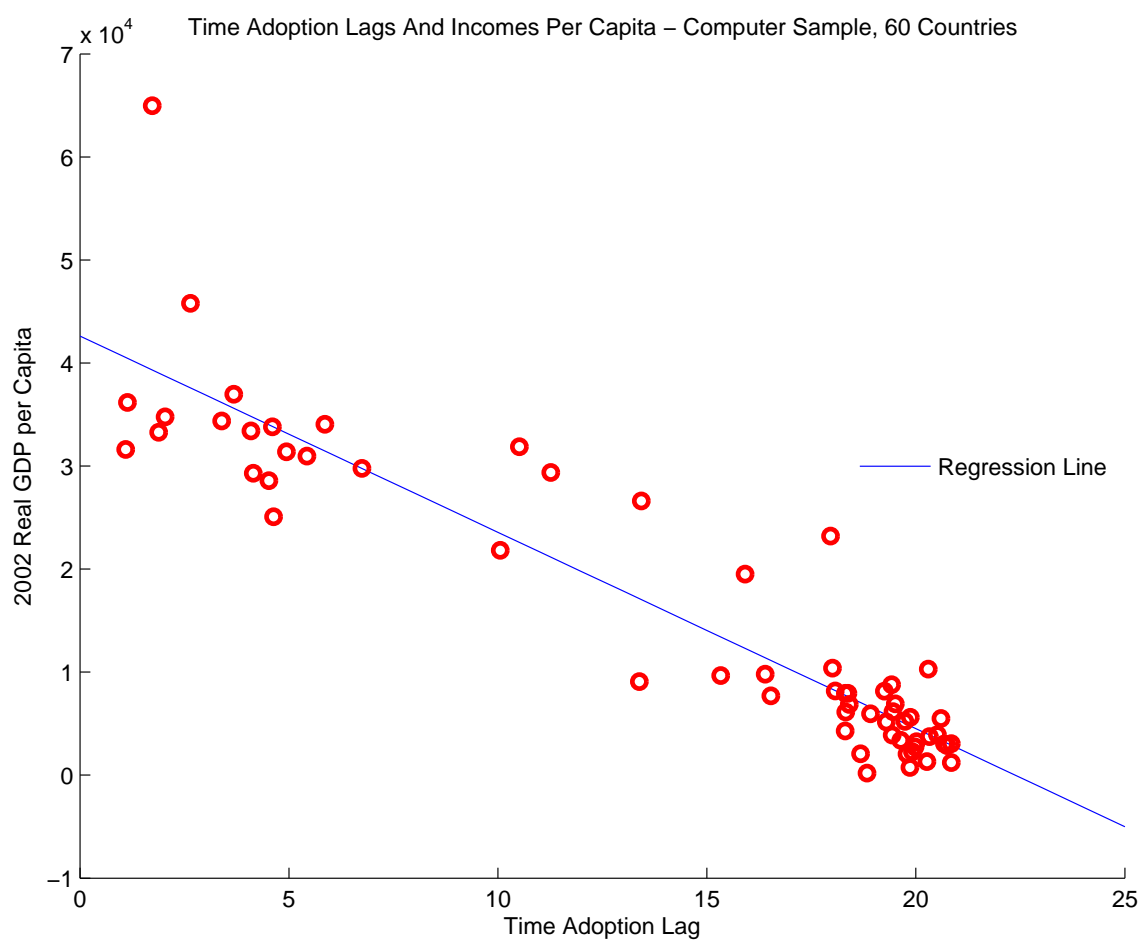


FIGURE 2.2: Time Adoption Lags and Incomes per Capita

Notes: *Time Adoption Lag* measures how many years ago the United States last had the usage level of computers per capita that each country had in the benchmark year 2002. Real GDP per capita is the PPP converted GDP per capita measured at 2005 constant prices.

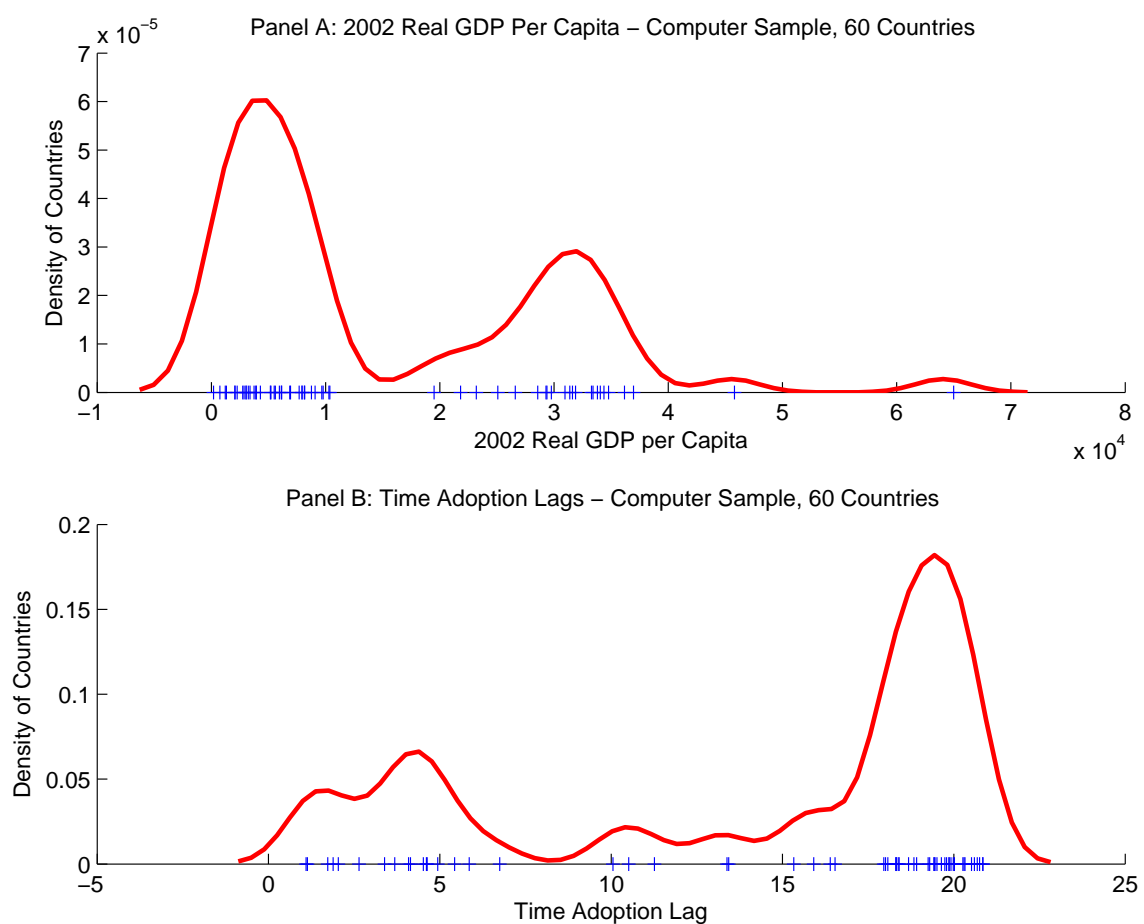


FIGURE 2.3: Cross-Country Distribution of Time Adoption Lags and Incomes Per Capita

Notes: *Time Adoption Lag* measures how many years ago the United States last had the usage level of computers per capita that each country had in the benchmark year 2002. Real GDP per capita is the PPP converted GDP per capita measured at 2005 constant prices.

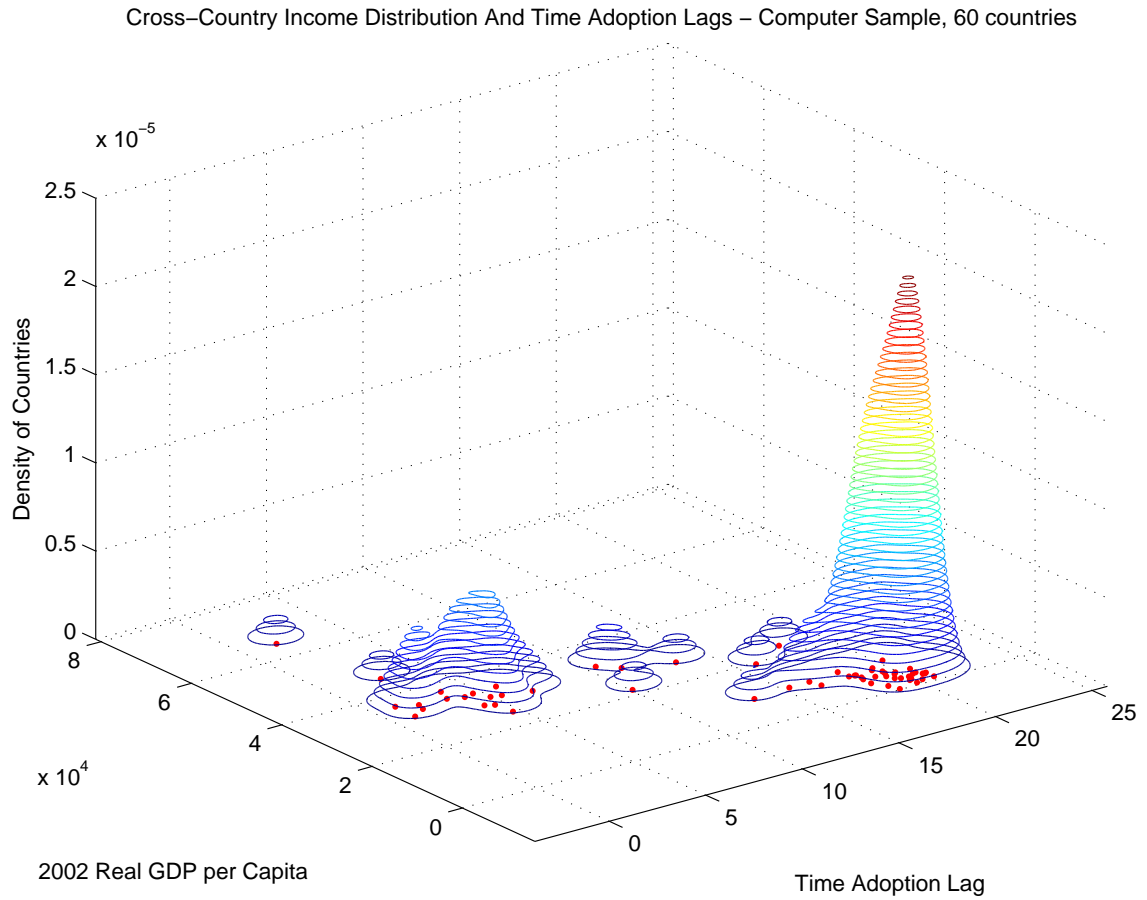


FIGURE 2.4: Cross-Country Joint Distribution of Time Adoption Lags and Incomes Per Capita

Notes: *Time Adoption Lag* measures how many years ago the United States last had the usage level of computers per capita that each country had in the benchmark year 2002. Real GDP per capita is the PPP converted GDP per capita measured at 2005 constant prices.

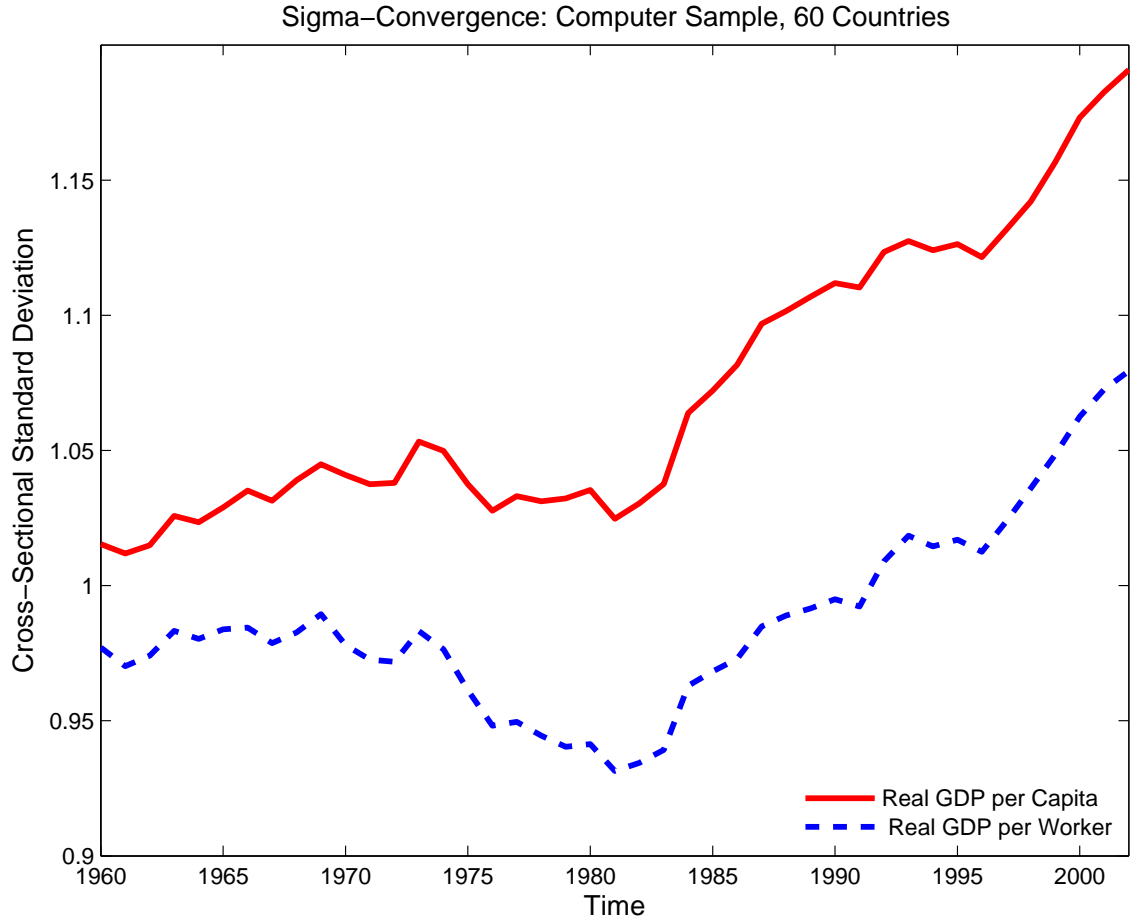


FIGURE 2.5: Sigma-Convergence — Computer Sample, 60 Countries

Notes: Real GDP per capita (per worker) is the PPP converted GDP per capita (per worker) measured at 2005 constant prices. On the y-axis, the cross-sectional standard deviation is calculated as $\sigma_t^{\text{cs}} = \left[\frac{1}{I} \sum_{i=1}^I (\ln(y_{it}) - \mu_t)^2 \right]^{1/2}$ with $\mu_t = \frac{1}{I} \sum_{i=1}^I \ln(y_{it})$, where y_{it} is the real GDP per capita (per worker) of country $i = 1, \dots, 60$ for $t = 1960, \dots, 2002$.

Table 2.2: Time Adoption Lag, Growth and Volatility — Descriptive Statistics

	Computers Sample (60 countries, 1973-2002)	Internet Sample (61 countries, 1983-2002)	Cell Phones Sample (68 countries, 1973-2002)
	(1)	(2)	(3)
<i>Panel A:</i>			
Mean <i>Lag</i>	14.131	6.651	9.450
Std. Dev. <i>Lag</i>	7.081	3.344	4.390
Lowest- <i>Lag</i> country	1.097 (Sweden)	0.294 (Australia)	1.350 (Chile)
Highest- <i>Lag</i> country	20.850 (Indonesia)	11.764 (Lesotho)	17.661 (Ethiopia)
<i>Panel B:</i>			
Mean <i>Growth</i>	0.016	0.015	0.010
Lowest- <i>Growth</i> country	-0.023 (Nicaragua)	-0.026 (Nicaragua)	-0.031 (Zambia)
Highest- <i>Growth</i> country	0.050 (Botswana)	0.052 (Botswana)	0.088 (Equatorial Guinea)
<i>Panel C:</i>			
Mean <i>Volatility</i>	0.044	0.037	0.058
Lowest- <i>Volatility</i> country	0.014 (France)	0.010 (Netherlands)	0.018 (Sri Lanka)
Highest- <i>Volatility</i> country	0.121 (Nicaragua)	0.107 (Guinea-Bissau)	0.229 (Equatorial Guinea)

Notes: *Lag* stands for Time Adoption Lag and measures how many years ago the U.S. last had the technology usage level that each country had in the benchmark year 2002. The variables *Growth* and the *Volatility* are respectively mean and standard deviation of per capita real GDP annual growth rates for each country and calculated over the relevant sample period. In the Cell Phones Sample, column (3), the second highest-*Growth* country is Botswana.

Table 2.3: The Relationship between Mean and Volatility of Growth

	Computers Sample (60 countries, 1973-2002)	Internet Sample (61 countries, 1983-2002)	Cell Phones Sample (68 countries, 1973-2002)
Independent Variable	(1)	(2)	(3)
<i>Volatility</i>	-0.229*** (0.078)	-0.277** (0.108)	-0.196*** (0.059)
Constant	0.027*** (0.003)	0.025*** (0.004)	0.020*** (0.004)
R^2	0.142	0.123	0.106
Observations	60	61	67

Notes: The dependent variable *Growth* and the explanatory *Volatility* are respectively mean and standard deviation of per capita real GDP annual growth rates calculated over the relevant sample period for each country. Heteroskedasticity robust standard errors in parentheses. Equatorial Guinea is excluded from the Cell phones sample because it is an influential observation. ***, ** indicate respectively statistical significance at the 1 and 5 percent level.

Table 2.4: Volatility and Time Adoption Lags — Computers per Capita

Independent Variable	Full Sample (60 countries, 1973-2002)			OECD Sample (24 countries, 1973-2002)		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Volatility</i>	155.025*** (27.134)		137.222*** (31.590)	272.463*** (71.549)		325.530*** (62.672)
<i>Growth</i>		-162.726*** (53.435)	-77.576 (57.089)		-208.031 (185.786)	-390.315** (139.156)
R^2	0.298	0.122	0.322	0.303	0.043	0.442
Observations	60	60	60	24	24	24

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the usage level of computers per capita that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates over the sample period 1973-2002 for each country. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, ** indicate respectively statistical significance at the 1 and 5 percent level.

Table 2.5: Volatility and Time Adoption Lags — Internet per Capita

Independent Variable	Full Sample (61 countries, 1983-2002)			OECD Sample (23 countries, 1983-2002)		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Volatility</i>	70.401*** (14.273)		52.844*** (16.264)	114.990** (48.381)		116.433** (52.889)
<i>Growth</i>		-86.939*** (19.737)	-63.391*** (19.751)		13.438 (35.841)	-7.128 (30.587)
R^2	0.207	0.196	0.298	0.251	0.004	0.252
Observations	61	61	61	23	23	23

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the usage level of internet per capita that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates over the sample period 1983-2002 for each country. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, ** indicate respectively statistical significance at the 1 and 5 percent level.

Table 2.6: Volatility and Time Adoption Lags — Cell Phones per Capita

Independent Variable	Full Sample (68 countries, 1973-2002)		
	(1)	(2)	(3)
<i>Volatility</i>	19.787 (13.633)		25.963* (13.647)
<i>Growth</i>		-94.344*** (30.229)	-99.686*** (21.941)
R^2	0.023	0.155	0.193
Observations	68	68	68

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the usage level of cell phones per capita that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates over the sample period 1973-2002 for each country. The OECD sample is not shown because it would include only four countries: time adoption lags for cell phones are censored because the U.S. never reaches the usage level that most OECD countries have in the benchmark year 2002. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, * indicate respectively statistical significance at the 1 and 10 percent level.

Table 2.7: Past Volatility Performance and Time Adoption Lags

	Computers (Full Sample, 60 countries)	Internet (Full Sample, 61 countries)	Cell Phones (Full Sample, 68 countries)
Independent Variable	(1)	(2)	(3)
<i>Volatility</i>	99.598*** (24.097)	57.960*** (10.372)	32.500*** (10.860)
<i>Growth</i>	-56.178* (33.365)	-40.694** (20.229)	-85.750*** (18.002)
R^2	0.156	0.308	0.264
Observations	60	61	68
Invention Year	1973	1983	1973

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the technology usage level that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates calculated over the sample period from 1960 to the invention year for each technology and country. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, **, * indicate respectively statistical significance at the 1, 5 and 10 percent level.

Table 2.8: Volatility and Quality-Adjusted Time Adoption Lags — Computers per Capita

Independent Variable	Full Sample (83 countries, 1973-2002)			OECD Sample (24 countries, 1973-2002)		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Volatility</i>	89.924*** (27.796)		94.686*** (19.226)	182.199*** (55.372)		218.498*** (50.259)
<i>Growth</i>		-133.819** (57.516)	-143.600*** (34.067)		-144.630 (117.346)	-266.981** (95.878)
R^2	0.165	0.117	0.299	0.324	0.050	0.479
Observations	83	83	83	24	24	24

Notes: The dependent variable *QA-Time Adoption Lag* measures how many years ago the U.S. last had the quality-adjusted usage level of computers per capita that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates over the sample period 1973-2002 for each country. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, ** indicate respectively statistical significance at the 1 and 5 percent level.

Table 2.9: Volatility and Technology Usage per Capita — Fixed-Effects Model

Independent Variable	Computers		Internet		Cell Phones
	Full Sample	OECD Sample	Full Sample	OECD Sample	Full Sample
	(1)	(2)	(4)	(5)	(6)
σ^{roll}	-0.784* (0.450)	-0.272 (1.430)	2.302 (2.959)	-2.687 (3.182)	1.694 (1.551)
g^{roll}	-0.988 (0.683)	0.275 (0.685)	2.750 (2.623)	-0.865 (1.600)	3.727 (1.751)
R^2	0.063	0.253	0.167	0.333	0.047
Countries	60	24	61	23	68
Observations	447	190	456	184	472

Notes: The dependent variable is the growth rate of the usage level of a specific technology. The explanatory variables g^{roll} and σ^{roll} are respectively the 5-year rolling mean and standard deviation of per capita real GDP annual growth rates over the sample period 1991-2002. Standard errors clustered by country in parentheses. Regressions include a constant and a set of year dummies. * indicates statistical significance at the 1 percent level.

Commodity Prices, Long-Run Growth and Fiscal Vulnerability (with Pietro Peretto)

3.1 Introduction

Oil prices and more generally commodity prices exhibit long-lasting declines and prolonged periods of rises. Figure 3.1 depicts the annual series of the real oil price (solid line) from 1861 to 2011 with a smooth trend (dotted line) estimated by fitting a second-order polynomial to the series.¹ The figure shows a striking U-shape pattern in the trend, which captures a period of roughly 100 years of declining prices replaced by 40 years of price increases. Notice also the 20 years pattern of steadily price declines from 1980 to 2000, which reverts roughly 10 years of price increases from 1970 to 1980. Similar patterns emerge in several other commodities, e.g., agricultural raw materials, metals, food and beverage. This figure is suggestive of long waves of price declines and raises. Overall, the evidence suggests that commodity price movements are large both at high and low frequency.²

¹ Data source: British Petroleum Statistical Review of World Energy Report 2012.

² See Jacks (2013) for an extensive treatment of long-run trends, medium-run cycles, and short-run boom/bust episodes in commodity prices.

It is also widely recognized that many real world economies, both developing and developed, depend for their exporting sector income on a narrow range of commodities. For these countries, long periods of commodity price declines are often associated with large tax revenue shortfalls and deteriorating public finances. Only few examples are the fiscal crisis occurred in Nigeria in 1991 and Kenya in the early 1980's when the prices of oil and coffee reverted to their previous levels prior to respectively the oil price boom at the time of the Iraqi invasion of Kuwait, and the 1976-1979 coffee price boom. Overall, the existing evidence suggests that extensive reliance on the tax base of the commodity-exporting sector makes the country's fiscal stance particularly vulnerable to variations in world commodity prices.³ We think of this phenomenon as *fiscal vulnerability*. The issue of fiscal dependence on commodity-linked revenues in commodity-rich countries has long been and still is a relevant matter for policy making. Quoting the 2013 Nigeria Economic Report by the World Bank, "*As oil revenues comprise 75 percent of budgetary revenues and 95 percent of exports in Nigeria, the effective management of the country's oil wealth is critical to stability and fiscal sustainability in the country.*" Hence, the current debate points to the *interaction* of fiscal policy with conditions in the commodity market as a mechanism particularly relevant for the economics of commodity-rich countries. Motivated by the empirical observations above and the renewed interest in the policy debate, this paper tackles, from a theoretical prospective, the following questions: absent fiscal considerations, how do commodity-exporting economies respond to commodity price changes? How does the economy dynamic response depend on the tax code in place? And how should governments react to world commodity price changes in the wake of tax revenue shortfalls? To answer these questions, we develop a Schumpeterian small open economy (SOE) model of endogenous growth.

³ See Sinnott (2009) for a detailed discussion of fiscal dependence on hydrocarbon revenues in Latin America and the Caribbean.

We study both the short- and long-run effects of commodity price changes and how fiscal policy interacts with the amplification and propagation of external shocks to these prices. In the spirit of the SOE tradition, we assume that commodity prices are taken parametrically by agents inside our model and determined in the world commodity market.

The paper studies the *joint* role of commodity prices and distortionary taxation in an environment where technological change is endogenous. We think endogenous growth theory is the natural framework to study these issues for two main reasons: first, there is now a large literature on the “curse of natural resources” which hints at very long-run effects of natural resources. However, as exemplified by the title of the *Journal of Economic Literature* survey paper by Van der Ploeg (2011), “Natural Resources: Curse or Blessing?”, the outcome of this research effort is far from conclusive, with mixed empirical evidence.⁴ In this latter regard, a model where steady-state growth is the endogenous equilibrium outcome of activities undertaken by economic agents, allows us to take seriously the notion of the curse of natural resources and provide theoretical predictions about conditions in the commodity market (and/or natural resources) and economic growth. Second, since historically governments have reacted differently in the wake of commodity prices booms and busts, we are interested in studying the effects of different tax policies, with a special focus on the differences between short- and long-run effects. Ultimately, our analysis provides insights on how to design welfare-enhancing tax policies for commodity-exporting countries.

Specifically, our theoretical framework is a Schumpeterian model of endogenous growth featuring both horizontal (expanding variety) and vertical (quality upgrading and/or cost reducing) innovation. Market structure is endogenous in that both *firm size* and the *mass* of firms are jointly determined in equilibrium. In fact, it

⁴ On a similar note see Gelb (1988).

is the interaction of the entry and quality margin of innovation—a *variety-quality frontier*—that drives the equilibrium dynamics of the model. A key property of our theoretical structure is that the model economy features different growth regimes, with transitions from one regime to the other being endogenous. We believe this is an appealing feature of the model since it allows us to derive predictions for economies that are at different stages of economic development. We provide results for both the transitional dynamics and the steady state of the model economy. A distinctive feature of the model is that long-run growth is independent of the scale of economic activity, i.e., there is no scale effect.⁵ This feature is essential for the purpose of the paper for at least two reasons. First, commodity price changes interact with the scale of the economy by the induced income effect. This implies that sterilization of the scale effect is needed to have a balanced growth path consistent with varying commodity prices. Second, analyses of fiscal policy under scale effects are subject to the Stokey and Rebelo (1995)’s critique. Models of endogenous growth exhibiting scale effects predict effects of fiscal policy that are too large compared to the available empirical evidence. Few examples of this evidence are Easterly and Rebelo (1993) and Mendoza et al. (1997) for cross-country growth regressions, Easterly et al. (1993) and Jones (1995) for time series evidence.

The results of the paper can be summarized as follows. 1) Commodity prices affect the short-run equilibrium dynamics of the model economy. Spending on manufacturing goods is increasing in the commodity price if the domestic demand for the commodity is inelastic. It is instead decreasing if the demand is elastic. The persistence of these short-run effects depends on the parameters of the model that determine the speed of reversion to the steady state. 2) Commodity price changes have no long-run growth effects, i.e., the steady-state growth rate of the model econ-

⁵ See Peretto (1998) for a detailed analysis of the mechanism driving the sterilization of the scale effect in this class of models.

omy is fully insulated from the conditions in the commodity market, that is, from commodity prices and endowments. This happens because endogenous entry makes the steady-state growth rate independent of the size of the manufacturing sector. It is exactly in this latter regard that the co-existence of the *entry* and *quality* margin of innovation plays a crucial role: the process of entry induces product proliferation which fragments the aggregate market into sub-markets whose size does not increase with the size of the manufacturing sector. The sterilization of this *market size effect* results in the sterilization of the *scale effect*, that is, the sterilization of the steady-state growth effects of parameters that drive the size of the economy. Finally, 3) the economy dynamic response to commodity price changes depends on both the structure of the tax code in place and on the policy response necessary to balance the government budget. Specifically, a distortionary tax on asset income (dividends plus capital gains) *amplifies* the effects of a given commodity price change and *slows down* the reversion to the steady state after a commodity price shock. Furthermore, if the government raises the tax rate on asset income in response to a resource revenue shortfall, then the commodity price has an indirect adverse effect on the steady-state growth rate of the model economy. This happens because tax rates affecting the equilibrium rate of return to cost reduction and entry have steady-state growth effects.⁶ Notice that, in this latter case, the negative steady-state growth effects are to be exclusively imputed to the (misguided) reaction of the government to a commodity price decrease, which, by changing the tax rate on asset income, it is distorting the effective return to innovation.

The paper is organized as follows. In Section 3.2, we discuss the setup of the model. In Section 3.3, we discuss the equilibrium of the market economy. Section 3.4 discusses the effects of commodity price changes. In Section 3.5, we introduce dis-

⁶ See Peretto (2003) for a previous discussion of level and steady-state growth effects of taxation in models of endogenous growth without scale effects.

tortionary taxation into the model. Section 3.6 contains a simple numerical exercise. Section 3.7 concludes.

3.2 The Model

3.2.1 Overview

We consider a small open economy (SOE) populated by a representative household that supplies labor services inelastically in a competitive labor market. The household faces a standard expenditure-saving decision such that it optimally chooses the path of expenditures (home and foreign goods) and savings by freely borrowing and lending in a competitive market for financial assets at the prevailing interest rate.⁷ The household's income consists of returns on asset holdings, labor income, profits and resource income. Resource income is the (constant) commodity endowment valued at the world commodity price.

The production side of the economy consists of three sectors: 1) consumption goods; 2) intermediate goods or manufacturing; and 3) materials. The consumption goods sector consists of a representative competitive firm which combines differentiated intermediate goods to produce an homogeneous final good. Upon entry, manufacturing firms combine labor services and materials to produce differentiated intermediate goods. They also engage in activities aimed to reduce their costs of production, and consequently, improve efficiency. Entry requires the payment of a sunk cost. Finally, materials are supplied by a separate competitive sector which demands as inputs labor services and the commodity paying the world commodity price. At this stage, there is no government sector, which we introduce into the model later in Section 3.5.

⁷ It is possible to think of our model economy as taking the world interest rate parametrically. Since the model has the property that the domestic interest rate jumps to its steady-state level, given by the domestic discount rate, as long as the SOE has the same discount rate as the rest of the world, the equilibrium discussed in the paper displays the same properties as an equilibrium with free financial flows.

The intermediate goods is the key sector of our model economy in that it is the engine of endogenous growth. Precisely, the economy starts out with a given range of intermediate goods, each supplied by one firm. Entrepreneurs compare the present value of profits from introducing a new good to the entry cost. They only target new product lines because entering an existing product line in Bertrand competition with the existing supplier leads to losses. Once in the market, firms devote labor to cost-reducing (or, equivalently, productivity enhancing) projects. As each firm strives to figure out how to improve efficiency, it contributes to the pool of public knowledge that benefits the future cost reduction activity of all firms. This allows the economy to grow at a constant rate in steady state, which is reached when entry stops and the economy settles into a stable industrial structure.

3.2.2 Households

The representative household maximizes lifetime utility

$$U(t) = \int_t^\infty e^{-\rho(s-t)} \log u(s) ds, \quad \rho > 0 \quad (3.1)$$

where

$$\log u = \varphi \log \left(\frac{Y_H}{P_H L} \right) + (1 - \varphi) \log \left(\frac{Y_F}{P_F L} \right), \quad 0 < \varphi < 1 \quad (3.2)$$

subject to the flow budget constraint

$$\dot{A} = rA + WL + \Pi_H + \Pi_M + p\Omega - Y_H - Y_F, \quad \Omega > 0 \quad (3.3)$$

where ρ is the discount rate, φ controls the degree of home bias in preferences, A is assets holding, r is the rate of return on financial assets, W is the wage, L is population size, which equals labor supply since there is no preference for leisure, Y_H is expenditure on a home consumption good whose price is P_H , and Y_F is expenditure on a foreign consumption good whose price is P_F . In addition to asset and labor

income, the household receives the dividends paid out by the producers of the home consumption good, Π_H , the dividends paid out by firms in the material sector, Π_M , and the revenues from sales of the domestic endowment of the commodity, Ω , at the world commodity price p . The solution to this problem consists of the optimal consumption-expenditure allocation rule

$$\varphi Y_F = (1 - \varphi) Y_H, \quad (3.4)$$

and the Euler equation governing saving behavior

$$r = r_A \equiv \rho + \frac{\dot{Y}_H}{Y_H} = \rho + \frac{\dot{Y}_F}{Y_F}. \quad (3.5)$$

3.2.3 Trade Structure

The foreign good is imported at the constant world price P_F . The economy can be either an importer or an exporter of the commodity. In the first case, it sells the home consumption good to buy the commodity in the world market; in the second case, it accepts the foreign consumption good as payment for its commodity exports. Only final goods and the commodity are tradable. The balanced trade condition, which is also the market clearing condition for the consumption good market, is $Y_H + Y_F + p(O - \Omega) = Y$, where Y is the aggregate value of production of the home consumption good. Using the consumption expenditure allocation rule (3.4), we can rewrite the balance trade condition as,

$$\frac{1}{\varphi} Y_H + p(O - \Omega) = Y, \quad (3.6)$$

where O is home use of the commodity.

3.2.4 The Consumption Good Sector

The home homogeneous consumption good is produced by a representative competitive firm with the technology

$$C_H = N^\chi \left[\int_0^N \frac{1}{N} X_i^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \chi > 0, \epsilon > 1 \quad (3.7)$$

where ϵ is the elasticity of product substitution, X_i is the quantity of the non-durable intermediate good i , and N is the mass of goods. We follow Ethier (1982) and separate the elasticity of substitution between intermediate goods from the degree of increasing returns to the variety of intermediate goods, χ . The final good producer maximizes,

$$\Pi_H = P_H C_H - \int_0^N P_i X_i di$$

subject to (3.7). This structure yields the demand curve for each intermediate good as

$$X_i = Y \frac{P_i^{-\epsilon}}{\int_0^N P_i^{1-\epsilon} di}, \quad (3.8)$$

where $Y = P_H C_H$. Because this sector is perfectly competitive, $\Pi_H = 0$.

3.2.5 The Intermediate Goods Sector

The typical firm produces one differentiated good with the technology

$$X_i = Z_i^\theta \cdot F(L_{X_i} - \phi, M_i), \quad 0 < \theta < 1, \phi > 0 \quad (3.9)$$

where X_i is output, L_{X_i} is production employment, ϕ is a fixed labor cost, M_i is use of materials, and Z_i^θ is the firm's total factor productivity (TFP), a function of the stock of firm-specific knowledge Z_i . The function $F(\cdot)$ is a standard production function homogeneous of degree one in its arguments. The associated total cost is,

$$W\phi + C_X(W, P_M) Z_i^{-\theta} \cdot X_i, \quad (3.10)$$

where $C_X(\cdot)$ is a standard unit-cost function homogeneous of degree one in its arguments. Hicks-neutral technological change internal to the firm shifts this function down. The elasticity of unit cost reduction with respect to firm-specific knowledge is the constant θ .

The firm accumulates knowledge according to the technology

$$\dot{Z}_i = \alpha K L_{Z_i}, \quad \alpha > 0 \quad (3.11)$$

where \dot{Z}_i is the flow of firm-specific knowledge generated by a project employing L_{Z_i} units of labor for an interval of time dt , and αK is the productivity of labor in such a project as determined by the exogenous parameter α and by the stock of public knowledge, K . Public knowledge accumulates as a result of spillovers.

When a firm generates a new idea to improve the production process, it also generates general-purpose knowledge which is not excludable and that other firms can exploit in their own research efforts. Firms appropriate the economic returns from firm-specific knowledge but cannot prevent others from using the general-purpose knowledge that spills over into the public domain. Formally, a project that produces \dot{Z}_i units of proprietary knowledge also generates \dot{Z}_i units of public knowledge. The productivity of research is determined by some combination of all the different sources of knowledge. A simple way of capturing this notion is to write

$$K = \int_0^N \frac{1}{N} Z_i di,$$

which says that the knowledge frontier is determined by the average knowledge of all firms.⁸

3.2.6 Materials

A representative competitive firm combines labor services, L_M , and commodities, O , to produce materials M , used as inputs in the manufacturing sector. The technology

⁸ For a detailed discussion of a spillovers function of this class, see Peretto and Smulders (2002).

is $M = G(L_M, O)$, where $G(\cdot)$ is a standard production function homogeneous of degree one in its arguments. The associated total cost is

$$C_M(W, p) M, \quad (3.12)$$

where $C_M(\cdot)$ is a standard unit-cost function homogeneous of degree one in the wage W and the commodity price p . This is the simplest way to model the materials sector for the purposes of this paper. Materials are produced with labor and the commodity purchased or sold at a given price in the world commodity market. The sector for materials competes for labor with the manufacturing sector. This captures the fundamental inter-sectoral allocation problem faced by this economy.

3.3 Agents' Behavior and Equilibrium Dynamics

This section constructs the equilibrium of the manufacturing sector. It then characterizes the equilibrium of the sector producing materials. Finally, it imposes general equilibrium conditions to determine the aggregate dynamics of the model economy.

3.3.1 The Manufacturing Sector

The typical intermediate firm maximizes the present discounted value of net cash flows,

$$V_i(t) = \int_t^\infty e^{-\int_t^s [r(v) + \delta] dv} \Pi_i(s) ds, \quad \delta > 0$$

where δ is a death shock. Using the cost function (3.10), instantaneous profits are

$$\Pi_i = [P_i - C_X(W, P_M) Z_i^{-\theta}] X_i - W\phi - WL_{Z_i},$$

where L_{Z_i} is labor devoted to cost-reducing projects. V_i is the value of the firm, the price of the ownership share of an equity holder. The firm maximizes V_i subject to the cost-reduction technology (3.11), the demand schedule (3.8), $Z_i(t) > 0$ (the

initial knowledge stock is given), $Z_j(t')$ for $t' \geq t$ and $j \neq i$ (the firm takes as given the rivals' knowledge accumulation paths), and $Z_j(t') \geq 0$ for $t' \geq t$ (knowledge accumulation is irreversible). The solution of this problem yields the (maximized) value of the firm given the time path of the number of firms.

To characterize entry, we follow Peretto and Connolly (2007) and assume that upon payment of a sunk cost $(\beta Y/N) \cdot W$, an entrepreneur can create a new firm that starts out its activity with productivity equal to the industry average.⁹ Once in the market, the new firm solves a problem identical to the one outlined above for the incumbent firm. A free entry equilibrium, therefore, requires $V_i = W \cdot (\beta Y/N)$.

The Appendix shows that the equilibrium thus defined is symmetric and is characterized by the factor demands:

$$WL_X = Y \frac{\epsilon - 1}{\epsilon} S_X^L + W \phi N; \quad (3.13)$$

$$P_M M = Y \frac{\epsilon - 1}{\epsilon} S_X^M, \quad (3.14)$$

where the shares of the firm's variable costs due to labor and materials are respectively:

$$S_X^L \equiv \frac{WL_{X_i}}{C_X(W, P_M) Z_i^{-\theta} X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log W};$$

$$S_X^M \equiv \frac{P_M M_i}{C_X(W, P_M) Z_i^{-\theta} X_i} = \frac{\partial \log C_X(W, P_M)}{\partial \log P_M}.$$

Note that $S_X^L + S_X^M = 1$. Associated to these factor demands are the rates of return to cost reduction and entry, respectively:

$$r = r_Z \equiv \frac{\alpha}{W} \left[\frac{Y}{\epsilon N} \theta (\epsilon - 1) - W \frac{L_Z}{N} \right] + \frac{\dot{W}}{W} - \delta; \quad (3.15)$$

⁹ See Peretto and Connolly (2007) for an interpretation of this assumption.

$$r = r_N \equiv \frac{N}{W\beta Y} \left[\frac{Y}{\epsilon N} - W\phi - W\frac{L_Z}{N} \right] + \frac{\dot{W}}{W} - \delta + \frac{\dot{Y}}{Y} - \frac{\dot{N}}{N}. \quad (3.16)$$

Neither the return to cost reduction in (3.15) nor the return to entry in (3.16) depend on factors related to the commodity market. Why is this the case? The production technology (3.9) yields a unit-cost function that depends only on input prices and is independent of the quantity produced and thus of inputs use. Since the optimal pricing rule features a constant markup over unit cost, the firm's gross-profit flow (revenues minus variable costs), $Y/\epsilon N$, is independent of input prices. Equations (3.15) and (3.16), then, capture the idea that investment decisions by incumbents and entrants do not respond directly to conditions in the commodity market because they are guided by the gross-profit flow. Conditions in the commodity market have an indirect effect through aggregate spending on intermediate goods, Y .

3.3.2 Materials

Given the cost function (3.12), competitive materials producers that purchase commodities at the given world price p operate along the infinitely elastic supply curve

$$P_M = C_M(W, p). \quad (3.17)$$

In equilibrium, then, materials production is given by (3.14) evaluated at the price P_M . Defining the commodity share in material costs as

$$S_M^O \equiv \frac{pO}{C_M(W, p)M} = \frac{\partial \log C_M(W, p)}{\partial \log p},$$

we can write the associated demands for labor and commodity as:

$$WL_M = M \frac{\partial C_M(W, p)}{\partial W} = Y \frac{\epsilon - 1}{\epsilon} S_X^M (1 - S_M^O); \quad (3.18)$$

and

$$pO = M \frac{\partial C_M(W, p)}{\partial p} = Y \frac{\epsilon - 1}{\epsilon} S_X^M S_M^O. \quad (3.19)$$

3.3.3 General Equilibrium

The model consists of the returns to saving (3.5), to cost reduction (3.15), and to entry (3.16), the labor demands in the manufacturing sector (3.13), materials (3.18), and the household's budget constraint (3.3).¹⁰ Assets market equilibrium requires equalization of all rates of return, $r = r_A = r_Z = r_N$, and that the value of the household's portfolio equal the value of the securities issued by firms, $A = NV = \beta WY$. We choose labor as the numeraire, i.e., $W \equiv 1$. A convenient implication of this normalization is that all expenditure terms are constant.

Proposition 12. *At any point in time, the value of home manufacturing production and the balanced trade condition, respectively, are:*

$$Y(p) = \frac{L}{1 - \xi(p) - \rho\beta}, \quad \text{with} \quad \xi(p) \equiv \frac{\epsilon - 1}{\epsilon} S_X^M(p) S_M^O(p); \quad (3.20)$$

$$\frac{1}{\varphi} Y_H(p) - p\Omega = Y(p) (1 - \xi(p)). \quad (3.21)$$

The associated expenditures on the home and foreign consumption goods, respectively, are:

$$Y_H(p) = \varphi \left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega \right]; \quad (3.22)$$

$$Y_F(p) = (1 - \varphi) \left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega \right]. \quad (3.23)$$

Because $Y_H(p)$ and $Y_F(p)$ are constant, the interest rate is $r = \rho$ at all times.

Proof. See the Appendix. □

¹⁰ The household's budget constraint (3.3) and the balance trade condition (3.6) imply the labor market clearing condition $L = L_N + L_X + L_Z + L_M$, where L_N is aggregate employment in entrepreneurial activity, $L_X + L_Z$ is aggregate employment in production and cost-reducing operations of existing firms, L_M is aggregate employment in materials producing firms. See the Appendix at the end of the paper for the derivations.

Given this structure of expenditures, the equilibrium dynamics are as follows.

Proposition 13. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate. The general equilibrium of the model reduces to the following piece-wise linear differential equation in the gross profit flow:*

$$\dot{x} = \begin{cases} \frac{\delta L\epsilon/N_0}{(1-\xi(p))^{-\frac{1}{\epsilon}}} & \text{if } \phi \leq x \leq x_N \\ \frac{\phi}{\beta\epsilon} - \left[\frac{1}{\beta\epsilon} - (\rho + \delta) \right] x & \text{if } x_N < x \leq x_Z \\ \frac{\phi - \frac{\rho + \delta}{\alpha}}{\beta\epsilon} - \left[\frac{1 - \theta(\epsilon - 1)}{\beta\epsilon} - (\rho + \delta) \right] x & \text{if } x > x_Z. \end{cases}$$

Assuming

$$\frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} > \frac{\rho + \delta}{\alpha\theta(\epsilon - 1)},$$

The economy converges to:

$$x^* = \frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)}. \quad (3.24)$$

The associated steady-state rate of cost-reduction is

$$\hat{Z}^* = \frac{(\phi\alpha - \rho - \delta)\theta(\epsilon - 1)}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)} - (\rho + \delta). \quad (3.25)$$

Proof. See the Appendix. □

Figure 3.2 illustrates the dynamics. Proposition 13 states a strong “**long-run commodity price super-neutrality**” result: the steady-state growth rate of the economy is independent of conditions in the commodity market and therefore of the commodity price p . In other words, the long-run economic *growth* performance of the model economy is insulated from external commodity price shocks. This happens because the sterilization of the market size effect through entry implies that steady-state growth does not depend on the size of the manufacturing sector and

therefore on the inter-sectoral allocation of labor. The resulting sterilization of the scale effect is a key property of the model coming from the interaction of the entry and quality margins of innovation. As new firms enter, the expansion in product variety fragments the aggregate market in sub-markets whose size does not increase with the size of the manufacturing sector. It is worth stressing that the same forces that yield the sterilization of the scale effect insulate the steady-state growth rate of the model economy from the commodity price.

3.3.4 Productivity, Utility and Welfare

Since the home consumption good sector is competitive, $P_H(p) = P_Y(p)$. Accordingly,

$$P_H(p) = N^{-\chi} \left[\frac{1}{N} \int_0^N (P_j(p))^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}} = N^{-\chi} Z^{-\theta} \left(\frac{\epsilon}{\epsilon-1} \right) C_X(1, C_M(1, p)),$$

where to simplify the notation, we define $c(p) \equiv C_X(1, C_M(1, p))$. We also define aggregate total factor productivity (TFP) as

$$T = N^\chi Z^\theta. \quad (3.26)$$

Accordingly,

$$\hat{T}(t) = \chi \hat{N}(t) + \theta \hat{Z}(t).$$

Using (3.25) in steady state this gives

$$\hat{T}^* = \theta \hat{Z}^* = \theta \left[\frac{(\phi\alpha - \rho - \delta)\theta(\epsilon-1)}{1 - \theta(\epsilon-1) - \beta\epsilon(\rho + \delta)} - (\rho + \delta) \right] \equiv g. \quad (3.27)$$

Observe how g is independent of conditions in the commodity market and of population size. In steady state, $x \equiv Y/\epsilon N$ is invariant to the commodity price p . We study the economy in the region $x(t) > x_Z$ and write the differential equation for x as

$$\dot{x} = \nu(x^* - x),$$

where

$$\nu \equiv \frac{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)}{\beta\epsilon} \quad \text{and} \quad x^* \equiv \frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\rho + \delta)}.$$

We thus work with the solution

$$x(t) = x_0 e^{-\nu t} + x^* (1 - e^{-\nu t}), \quad (3.28)$$

where x_0 is the initial condition. The following states the key result.

Proposition 14. *Consider an economy starting at time $t = 0$ with initial condition x_0 . At any time $t > 0$ the log of TFP is*

$$\log T(t) = \log(Z_0^\theta N_0^\chi) + gt + \left(\frac{\gamma}{\nu} + \chi\right) \Delta (1 - e^{-\nu t}), \quad (3.29)$$

where

$$\Delta \equiv \frac{x_0}{x^*} - 1.$$

The instantaneous utility flow is

$$\log u(t) = \log \varphi \left(\frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} + \frac{p\Omega}{L} \right) - \varphi \log c(p) + \varphi gt + \varphi \left(\frac{\gamma}{\nu} + \chi \right) \Delta (1 - e^{-\nu t}). \quad (3.30)$$

The resulting level of welfare is

$$U(0) = \frac{1}{\rho} \left[\log \varphi \left(\frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} + \frac{p\Omega}{L} \right) - \varphi \log c(p) + \frac{\varphi g}{\rho} + \frac{\varphi \left(\frac{\gamma}{\nu} + \chi \right) \Delta}{\rho + \nu} \right]. \quad (3.31)$$

Proof. See the Appendix. □

This structure identifies three main effects: 1) the *windfall effect* through $p\Omega$; 2) the *cost of living effect* through $c(p)$; and 3) the *curse or blessing effect* through

g and the transitional dynamics associated to Δ , the initial displacement from the steady state. These effects drive welfare as follows. The first two terms in (3.31) capture the role of steady-state utility calculated holding technology, T , constant; the third term captures the role of steady-state growth, g ; the fourth term is the contribution from the acceleration/deceleration of TFP growth along the transition.

The first two *static* components capture forces that the literature has discussed at length. An economy with a positive endowment of a commodity that sells for a higher price experiences a windfall. In our model this shows up as a rise in commodity income, which, given our assumptions, is formally equivalent to a lump-sum transfer from abroad. The cost of living effect is due to the fact that the economy uses the commodity for home production and, therefore, an increase in the world commodity price works its way through the home vertical structure of production – from upstream materials production to downstream manufacturing – and shows up as a higher price of the home consumption good.

The last two *dynamic* components capture forces that are the focus of modern endogenous growth theory. While the role of steady-state growth is well understood, this model allows us to investigate in detail the less studied role of the transitional dynamics. The reason is that we have a closed-form solution for the model's dynamics. Specifically, the fourth effect runs through the TFP operator in (3.29), which has two transitional components: the first is the cumulated gain/loss from the acceleration/deceleration of the rate of cost reduction; the second is the cumulated gain/loss from the acceleration/deceleration of product variety expansion. What these accelerations/decelerations do, is amplify the change in manufacturing expenditure due to a change in the commodity price. We discuss this mechanism in the next section.

3.4 The Dynamic Effects of World Price Shocks

In this section, we analyze the effects of commodity price changes on the transitional dynamics of the model economy. In Section 3.3.3 above, we argued that the steady-state growth rate is independent of the conditions in the commodity market. In a nutshell, the long-run economic *growth* performance of the model economy is insulated from external price shocks. However, conditions in the commodity market still matter for the short-run and the transition to the steady state.

An important building block of our theory is that the commodity, which is a fixed endowment of the economy, is used as input into production of materials. Hence, the demand of the commodity is endogenous and it responds to variations in the exogenous price p . This implies that the status of commodity importer or exporter is determined within the model as a function of the endowment Ω , price p , technological properties subsumed in the term $\xi(p)$, and other relevant parameters of the model. The following proposition characterizes the commodity-exporting or commodity-importing regions, and the effects of commodity price changes on manufacturing expenditure. We begin with the effect on manufacturing activity.

Lemma 15. *Let:*

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M};$$

$$\epsilon_M^R \equiv -\frac{\partial \log R}{\partial \log p} = 1 - \frac{\partial \log S_M^R}{\partial \log p} = 1 - \frac{\partial S_M^R}{\partial p} \frac{p}{S_M^R}.$$

Then,

$$\xi'(p) = \frac{\epsilon - 1}{\epsilon} \frac{\partial (S_M^O(p) S_X^M(p))}{\partial p} = \Gamma(p) \frac{\xi(p)}{p},$$

where

$$\Gamma(p) \equiv (1 - \epsilon_X^M(p)) S_M^O(p) + 1 - \epsilon_M^O(p). \quad (3.32)$$

Proof. See the Appendix. □

The key object in this lemma is the function $\Gamma(p)$, which is the elasticity of $\xi(p) \equiv \frac{\epsilon-1}{\epsilon} S_M^O(p) S_X^M(p)$ with respect to p . According to (3.19), therefore, it is the elasticity of the home demand for the commodity with respect to the world price, holding constant manufacturing expenditure. It thus captures the partial equilibrium effects of price changes in the commodity and materials markets for given market size. Differentiating (3.20), rearranging terms and using (3.19) yields

$$\frac{d \log Y(p)}{dp} = \frac{\xi'(p)}{1 - \xi(p) - \beta\rho} = \Gamma(p),$$

which says that the effect of changes in the resource price on expenditure on manufacturing goods depends on the overall pattern of substitution that is reflected in the price elasticities of materials and commodity demand and in the commodity share of materials production costs. The following proposition states the results formally.

Proposition 16. *Depending on the properties of the function $\Gamma(p)$, there are four cases.*

1. **Global complementarity.** *Suppose that $\Gamma(p) > 0$ for all p . Then, manufacturing expenditure $Y(p)$ in (3.20) is a monotonically increasing function of p .*
2. **Cobb-Douglas-like economy.** *Suppose that $\Gamma(p) = 0$ for all p . This occurs when S_M^O and S_X^M are exogenous constants. Then, manufacturing expenditure $Y(p)$ in (3.20) is independent of p .*
3. **Global substitution.** *Suppose that $\Gamma(p) < 0$ for all p . Then, manufacturing expenditure $Y(p)$ in (3.20) is a monotonically decreasing function of p .*

4. **Endogenous switch from complementarity to substitution.** Suppose that there exists a price p^v where $\Gamma(p)$ changes sign, from positive to negative. Then, manufacturing expenditure $Y(p)$ in (3.20) is a hump-shaped function of p with a maximum at p^v .

The Cobb-Douglas-like case is quite common in the literature as it occurs when both technologies are Cobb-Douglas with $\epsilon_X^M = \epsilon_M^O = 1$. We mention this case but not discuss it further since it is a knife-edge specification in which the world commodity price has no effect on home manufacturing activity. Proposition 16 says that the sign of the effect of the world commodity price on manufacturing activity depends on the substitution possibilities between labor and materials in manufacturing and between labor and the commodity in materials production. The most interesting case is when $\Gamma(p)$ changes sign at p^v and the model generates the endogenous switch from substitution to complementarity. We focus on this specification because it nests the two cases of monotonic effect of the price as we let $p^v \rightarrow 0$ or $p^v \rightarrow \infty$.

This analysis says that a commodity price boom raises home manufacturing activity when the economy exhibits overall complementarity between labor and the commodity. When the economy exhibits overall substitution, instead, a price boom results into a contraction of manufacturing activity. What about import/export behavior? We state the key result as follows.

Proposition 17. *The economy is an exporter of the commodity when*

$$\frac{\Omega}{L} > \frac{1}{1 - \xi(p) - \beta\rho} \frac{\xi(p)}{p}. \quad (3.33)$$

Proposition 17 contains several important results. 1) It provides an intuitive notion of “**commodity supply dependence**” that captures the traditional view that

a country that imports a key commodity is *dependent on foreign supply* and thus subject to external shocks. In our model, for a given commodity price $p > 0$, there exists a threshold of the endowment ratio Ω/L such that for Ω/L below this threshold the economy is an *importer*, i.e., $O > \Omega$, and for Ω/L above it the economy is an *exporter*, i.e., $O < \Omega$. An extreme case of dependence is when $\Omega = 0$ such that by assumption the country must import the commodity. 2) Another way to see the link between the commodity price and the importer/exporter status is to note that, for a given relative endowment Ω/L , there exists a price threshold p^d such that for $p < p^d$ the economy is an importer whereas for $p > p^d$ the economy is an exporter. 3) The proposition also provides an intuitive notion of “**dynamic commodity vulnerability**” captured by the property that $\Gamma(p) \gtrless 0$ determines whether the economy gains or loses from its dynamic response to a higher commodity price. Specifically, if $\Gamma(p) > 0$ home manufacturing expenditure increases in response to a commodity price increase. If $\Gamma(p) < 0$, instead, home manufacturing expenditure decreases with a commodity price increase. How persistent these effects are depends on the parameters of the model. 4) As stated earlier, we focus on the case in which the upstream materials sector and the downstream manufacturing sector have opposite substitutability/complementarity properties. Specifically, we assume that materials production exhibits labor-commodity complementarity while manufacturing exhibits labor-materials substitution. Accordingly, there exists a threshold of the commodity price where the economy switches from overall complementarity to overall substitutability. More precisely, there exists a threshold price p^v such that $\Gamma(p) < 0$ for $p < p^v$ and $\Gamma(p) > 0$ for $p > p^v$. The reason is that when p is low the cost share $S_M^O(p)$ is small and $\Gamma(p)$ is dominated by the term $1 - \epsilon_M^O(p)$, which is positive since complementarity implies $\epsilon_M^O(p) < 1$. In contrast, when p is high, the cost share $S_M^O(p)$ is large and $\Gamma(p)$ is dominated by the term $1 - \epsilon_X^M(p)$, which is negative since substitution implies $\epsilon_X^M(p) > 1$.

Definition 18. *An economy is dependent on the world commodity supply if $\Omega < O$, that is, if it consumes more of the commodity than it has. An economy is vulnerable to increases in the world commodity price p if its demand for the commodity is elastic.*

Notice that dependence and vulnerability are not the same. The reason is that home demand is endogenous and adjusts to the world price of the commodity. Figure 3.3 illustrates the determination of the threshold prices p^d and p^v . The threshold price p^d is increasing in the endowment ratio L/Ω and goes to infinity as $L/\Omega \rightarrow \infty$. Thus, economies with zero endowment, $\Omega = 0$, are dependent for all p . Interestingly, an economy with a positive endowment can gain from a higher world commodity price even if it is dependent. The reason is that the revenues from sales of the endowment Ω go up one-for-one with p while import costs go up less than linearly since home commodity consumption O responds negatively to p . Intuitively, this *specialization effect* is stronger the more elastic is home commodity demand. More importantly, however, what potentially matters most for welfare is not whether the country experiences an improvement in its commodity trade balance, but whether it is dynamically vulnerable in the sense defined above. And for dynamic vulnerability, elastic commodity demand is bad news. The reason is that the contraction of home commodity demand is just the other side of the contraction of manufacturing activity associated to the specialization effect. The Schumpeterian mechanism at the heart of the model amplifies such a contraction – the instantaneous fall in $Y(p)$ – into a deceleration of the rate of TFP growth. The economy eventually reverts to the steady-state growth rate g , but the temporary deceleration has a potentially substantial negative effect on welfare.

With these considerations in mind, now imagine a *permanent* change in the com-

modity price. For $p' > p$ we can write

$$\Delta \equiv \frac{x_0}{x^*} - 1 = \frac{Y(p')/\epsilon N(p)}{Y(p')/\epsilon N(p')} - 1.$$

This is the percentage displacement of the state variable x from its steady state that occurs at time 0 when the commodity price jumps up from p to p' . The numerator is the value of profitability holding constant the mass of firms; the denominator is the value of profitability at the end of the transition, when the mass of firms has fully adjusted to the new market size. Consider the case of a vulnerable exporter, that is, an exporter of the commodity with $\Gamma(p) < 0$.

Figure 3.4 illustrates the path of $\log u(t; p')$. On impact, technology T is pre-determined and does not jump, while the price index spike and the windfall effect work in opposite directions and yield an initial jump in consumption that has an ambiguous sign. Thereafter, the transitional effects of endogenous TFP take over. The permanent fall in Y produces a slowdown of TFP growth due to a slowdown of entry and a reduction in cost-reducing activity internal to the firm. It is apparent then, that the world commodity price boom benefits this economy if and only if the windfall effect through $p\Omega$ is large enough to compensate for the *cost of living effect* through $c(p)$ and the *curse effect* through $\Delta < 0$. Our explicit solution (3.31) in Proposition 14 shows how the model's parameters determine the weights of these effects.

3.5 Fiscal Policy

In this section, we introduce distortionary taxation into the model and study how the interaction of taxes and conditions in the commodity market shapes the dynamic response of the model economy to unexpected variations in the commodity price. The government taxes asset income, rA , at rate τ_A , commodity income, $p\Omega$, at rate τ_Ω

and consumption expenditures, $Y_H + Y_F$, at rate τ_C . We consider the policy scenario in which all tax proceeds are rebated in lump-sum form. Hence, we abstract from income effects and focus exclusively on the pure distortions introduced by *ad-valorem* taxes. We compare both steady-state growth rates and transitional dynamics.

The household budget constraint now reads,

$$\dot{A} = (1 - \tau_A)rA + WL + \Pi_H + \Pi_M + (1 - \tau_\Omega)p\Omega - (1 + \tau_C)(Y_H + Y_F) + R,$$

where $R = \tau_A rA + \tau_C(Y_H + Y_F) + \tau_\Omega p\Omega$ are the proceeds collected from income taxation.¹¹ Specifically, $\tau_A rA$ and $\tau_C(Y_H + Y_F)$ are revenues from taxing respectively asset income and consumption expenditures and $\tau_\Omega p\Omega$ are commodity-linked revenues. We abstract from taxation of profits. In our context, this choice is innocuous for two reasons: first, from the prospective of the government budget constraint, profits taxation does not generate any revenues. Since home consumption goods (H) and materials (M) sectors are both competitive, in equilibrium $\Pi_H = \Pi_M = 0$; second, in the current formulation, a positive tax rate on profits would have no distortionary effect on the equilibrium optimal allocations. We also abstract from labor income taxation since we assume labor services are inelastically supplied by the representative household. Taxing labor income would generate revenues to the government. However, this latter margin is irrelevant since we focus on the scenario in which tax revenues are lump-sum rebated to the household.

Because the tax rate on consumption expenditures τ_C is constant over time, the optimal expenditure sharing rule remains unaltered such that $\varphi Y_F = (1 - \varphi)Y_H$. However, the Euler equation governing the intertemporal consumption-saving decision changes,

$$(1 - \tau_A)r = r_A \equiv \rho + \frac{\dot{Y}_H}{Y_H} = \rho + \frac{\dot{Y}_F}{Y_F}. \quad (3.34)$$

¹¹ In this formulation of the tax code, the tax base for τ_A is asset income, that is, the sum of dividends and capital gains. This is equivalent to tax dividends and capital gains at the same rate.

Intuitively, the tax rate on asset income τ_A decreases the rate of return to savings. Since in equilibrium both $\dot{Y}_H/Y_H = \dot{Y}_F/Y_F = 0$, Equation (3.34) implies $(1-\tau_A)r = \rho$. For notational simplicity, let $\tilde{\rho} \equiv \rho/(1-\tau_A)$ denote the *effective* discount rate, and notice that an increase in the tax rate τ_A leads to an increase in $\tilde{\rho}$, such that the effective discount rate is actually increasing in the tax on asset income.

3.5.1 Growth Effects of Taxation

This section focuses on the effects of taxation on the steady-state growth rate and the transitional dynamics of the model.

Proposition 19. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate. The general equilibrium of the model with taxes reduces to the following piece-wise linear differential equation in the gross profit flow:*

$$\dot{x} = \begin{cases} \frac{\delta L\epsilon/N_0}{(1-\xi(p))-\frac{1}{\epsilon}} & \text{if } \phi \leq x \leq x_N \\ \frac{\phi}{\beta\epsilon} - \left[\frac{1}{\beta\epsilon} - (\tilde{\rho} + \delta) \right] x & \text{if } x_N < x \leq x_Z \\ \frac{\phi - \frac{\tilde{\rho} + \delta}{\alpha}}{\beta\epsilon} - \left[\frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\tilde{\rho} + \delta) \right] x & \text{if } x > x_Z. \end{cases}$$

Assuming

$$\frac{\phi - (\tilde{\rho} + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\tilde{\rho} + \delta)} > \frac{\tilde{\rho} + \delta}{\alpha\theta(\epsilon - 1)},$$

the economy with taxes converges to

$$x^* = \frac{\phi - (\tilde{\rho} + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \beta\epsilon(\tilde{\rho} + \delta)}.$$

The associated steady-state rate of cost-reduction is

$$\hat{Z}^* = \frac{(\phi\alpha - \tilde{\rho} - \delta)\theta(\epsilon - 1)}{1 - \theta(\epsilon - 1) - \beta\epsilon(\tilde{\rho} + \delta)} - (\tilde{\rho} + \delta),$$

which is decreasing in the tax rate on asset income τ_A .

Proof. The proof follows the same steps of Proposition 13 with ρ replaced by $\tilde{\rho}$. The comparative statics with respect to τ_A follows by simply differentiating the steady-state growth rate of quality innovation with respect to $\tilde{\rho}$ and recognizing that $\tilde{\rho}$ is increasing in the tax rate τ_A . \square

Proposition 19 provides two important results. 1) Asset income taxation has an adverse effect on the steady-state growth rate of quality innovation, which is the only driver of long-run growth in TFP. This happens because the asset income tax distorts the return to savings, and through the no arbitrage condition $r = r_A = r_Z = r_N$, it negatively affects the effective return to innovation. 2) Tax rates on consumption expenditures and commodity income have instead no effect on the steady-state growth rate of the model economy. This latter result is not special to the case with lump-sum tax rebates but it applies generally also with unproductive government spending.

The following lemma characterizes the effects of taxation on the dynamics of the gross profit flow x .

Lemma 20. *Let $x \equiv Y/\epsilon N$ denote the gross profit rate, and consider the solution to the general equilibrium of the model $x(t) = x_0 e^{-\tilde{\nu}t} + x^*(1 - e^{-\tilde{\nu}t})$ where x_0 is the initial condition, x^* is the steady state value of x , and the eigenvalue of the differential equation for x is*

$$\tilde{\nu} = \begin{cases} \frac{1}{\beta\epsilon} - (\tilde{\rho} + \delta) \equiv \tilde{\nu}_N & \text{if } x_N < x \leq x_Z \\ \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - (\tilde{\rho} + \delta) \equiv \tilde{\nu}_Z & \text{if } x > x_Z. \end{cases}$$

The eigenvalue $\tilde{\nu}$ is decreasing in the tax rate on asset income τ_A .

Proof. The comparative statics with respect to τ_A follows by simply differentiating the eigenvalue $\tilde{\nu}$ with respect to $\tilde{\rho}$ and realizing that $\tilde{\rho}$ is increasing in the tax rate τ_A . \square

Lemma 20 provides an important result. A positive tax rate on asset income slows down the transitional dynamics towards the steady state. This happens because the tax rate on asset income τ_A reduces the eigenvalue of the dynamical system which is the only driver of the transitional dynamics of the model.

3.5.2 Level Effects of Taxation

This section focuses on the level effects of taxation. The following proposition characterizes the effects of taxation on the level of manufacturing expenditure.

Proposition 21. *Under lump-sum rebate of tax proceeds, $R = \tau_A r A + \tau_\Omega p \Omega + \tau_C (Y_H + Y_F)$, manufacturing expenditure is*

$$Y(p) = \frac{L}{1 - \xi(p) - \beta \tilde{\rho}}.$$

Manufacturing expenditure is increasing in the tax rate on asset income, τ_A , i.e.,

$$\partial \log Y(p) / \partial \tau_A > 0.$$

Moreover, a positive tax on asset income, $\tau_A > 0$, acts as an automatic amplifier of commodity price changes,

$$\frac{d^2 \log Y(p)}{dp d\tau_A} = \begin{cases} \frac{\rho \beta \xi'(p) / (1 - \tau_A)^2}{[1 - \xi(p) - \beta \tilde{\rho}]^2} \geq 0 & \text{if } \xi'(p) \geq 0 \\ \frac{\rho \beta \xi'(p) / (1 - \tau_A)^2}{[1 - \xi(p) - \beta \tilde{\rho}]^2} \leq 0 & \text{if } \xi'(p) \leq 0. \end{cases}$$

Proof. The proof of the proposition follows the same steps of Proposition 12. □

Proposition 21 contains three results. 1) Manufacturing expenditure is increasing in the tax rate on asset income τ_A . This happens because a higher tax on asset income leads a reallocation from savings towards consumption expenditures which in equilibrium translates into larger expenditure on manufacturing goods. 2) A positive tax rate on asset income, $\tau_A > 0$, is an *automatic amplifier* of commodity

price changes. 3) Only the tax rate τ_A enters the determination of manufacturing expenditures. This happens because both tax rates on consumption expenditures and commodity income are, for different reasons, not distortionary. The tax rate τ_C is not distortionary because it is by assumption constant over time, as such it does not interfere with the intertemporal consumption allocations. Moreover, since we assume an inelastic supply of labor services, τ_C has also no intratemporal distortionary effect on the consumption-leisure optimal allocation. The tax rate levied on commodity income τ_Ω is not distortionary because the commodity is in fixed supply and the price is assumed constant and exogenous. Recall also that by focusing on the case of lump-sum rebates, we abstract from potential income effects of taxation.

3.5.3 Further Discussion

The combined of Propositions 19 and 21, and Lemma 20 conveys the main message of this section: asset income taxation has both *level* and *growth* effects. Furthermore, it acts as an *automatic amplifier* of commodity price changes and *slows down* the transitional dynamics of the model.

From a more general viewpoint, this section also suggests that in this class of models, the effects of taxation fall in two separate categories: tax instruments that have only *level* effects and no *growth effects*; tax instruments that have both *level* and *growth* effects.¹² By distorting the rate of return through the no arbitrage condition $r = r_A = r_Z = r_N$, the tax rate on asset income τ_A affects the equilibrium steady-state growth rate of the model economy because it alters the incentives to innovation. To this category belong not only taxes on asset income, but also any other tax that creates a wedge in the Euler equation, e.g., time-varying consumption taxes. Tax rates not interfering with the return to savings and innovation have no steady-state *growth* effects but have *level* effects on the endogenous variables of the

¹² See Peretto (2003) for an early discussion of this argument.

model. To this category belong commodity income taxes, constant consumption taxes, and labor income taxes when tax revenues are no longer lump-sum rebated to the household. In our context, consumption and commodity income taxes have no level effects because their potential income effect is neutralized by the lump-sum transfer to the household.

3.6 Numerical Analysis

To further understand the mechanics of the model and how the tax code in place affects the dynamic properties of our model economy, we conduct a simple numerical exercise. We assign numerical values to the relevant parameters of the model and let the tax rate on asset income τ_A take values that range from 20 to 50 percent. Notably, this wide range of tax rates is consistent with the available evidence on cross-country capital income tax rates provided by Mendoza et al. (1994).

3.6.1 Calibration

One period is one year. Table 3.1 contains the baseline parameter values that are kept constant over the following analysis.

We set $\epsilon = 4.33$ to match a price markup of 30 percent. Overall, the available evidence for the U.S. provides estimates of markups in value added data that range from 1.2 to 1.4.¹³ Hence, we target a markup in the manufacturing sector of $\mu = \epsilon/(\epsilon - 1) = 1.3$ that is at the middle of the available range of estimates. The condition for a symmetric equilibrium, $\theta(\epsilon - 1) < 1$, imposes a restriction on the calibration of θ , i.e., $\theta \in (0, 1/(\epsilon - 1))$. Furthermore, given the calibrated value of $\epsilon = 4.33$, we have an upper bound on θ such that $\theta \in (0, 0.3)$. Since we have no reference value guiding our choice, we set $\theta = 0.15$ at the middle of the possible range. The death

¹³ See Hall (1988), Morrison (1992), Norrbin (1993), Roeger (1995), and Basu and Fernald (1997, 2001).

rate is set to $\delta = 0.035$ to match the average closing rate of establishments in the U.S. manufacturing sector for 1992-2012. Data for closing establishments are from the Business Employment Dynamics (BED) survey of the Bureau of Labor Statistics (BLS). The requirement of positive eigenvalues over all the state space of the model imposes a restriction on the calibration of the entry cost β . Specifically, $\tilde{\nu}_Z > 0$ implies $\beta \in \left(0, \frac{1-\theta(\epsilon-1)}{\epsilon(\bar{\rho}+\delta)}\right)$. Notice that $\tilde{\nu}_Z = \tilde{\nu}_N - \theta(\epsilon-1)/\beta\epsilon < \tilde{\nu}_N$ guarantees that the restriction on β is a sufficient condition to have both eigenvalues always greater than zero. We normalize the entry cost at $\beta = 1$, which is within the set identified by the above restrictions. Finally, the time discount rate is set to the conventional value of 2 percent.

In Figure 3.5 below, we consider a wide spectrum of tax rates that ranges from 20 to 50 percent. Importantly, such a variation in tax rates—20% to 50%—is consistent with empirical estimates of capital income tax rates. For example, look at the updated estimates of effective tax rates for a sample of seven OECD countries over the period 1965-1996, calculated with the method proposed in Mendoza et al. (1994) and available on Mendoza’s website.

Table 3.2 reports the updated estimates of capital income tax rates for two sample years. In the data, capital income tax rates display large cross-country variation. In 1980, they range from a minimum of 20 percent for Italy to a maximum of 64.32 percent for the U.K. In 1996, from approximatively a 24 percent for Germany to 51 percent for Canada.

3.6.2 *Dynamic Response to a "Profit Rate Shock"*

In this section, we compute the dynamic response of the gross profit rate, $x \equiv Y(p)/\epsilon N(p)$, to a shock that temporarily displaces x from its steady-state value. In other words, we force the model to be in transition and study how the reversion to the original steady state depends on the value of the asset income tax τ_A . To this aim, we

keep the relevant parameters of the model fixed and vary the tax rate on asset income. Recall that the gross profit rate x is the key state variable of the model regulating the incentives to innovate and hence driving the relevant equilibrium dynamics.

Figure 3.5 plots the time path of

$$\frac{x(t)}{x^*} - 1 = \Delta e^{-\tilde{\nu}t}$$

where $x(t) = x_0 e^{-\tilde{\nu}t} + x^* (1 - e^{-\tilde{\nu}t})$, the eigenvalue of the differential equation for x is

$$\tilde{\nu} = \begin{cases} \frac{1}{\beta\epsilon} - \left(\frac{\rho}{1-\tau_A} + \delta \right) \equiv \tilde{\nu}_N & \text{if } x_N < x \leq x_Z \\ \frac{1-\theta(\epsilon-1)}{\beta\epsilon} - \left(\frac{\rho}{1-\tau_A} + \delta \right) \equiv \tilde{\nu}_Z & \text{if } x > x_Z \end{cases}$$

and the initial percentage displacement from the steady state is $\Delta = \frac{x_0}{x^*} - 1$ —we call this a “*profit rate shock*.” Given an initial displacement of $\Delta = 1\%$, we assess how the speed of reversion to the steady state differs at different levels of asset income taxation. Since the equilibrium gross profit flow x follows a linear differential equation, the speed of reversion to the steady state is governed by the magnitude of the eigenvalue $\tilde{\nu}$, which depends on the tax rate τ_A and other parameters. Precisely, the eigenvalue $\tilde{\nu}$ is decreasing in τ_A , i.e., higher asset income taxation leads to a slower reversion to the steady state for a given displacement Δ . Furthermore, $\tilde{\nu}_Z = \tilde{\nu}_N - \theta(\epsilon - 1)/\beta\epsilon < \tilde{\nu}_N$ such that the dynamics in the “entry and quality regime” (i.e. for $x > x_Z$) are slower than those in the “entry only regime” (i.e. for $x_N < x \leq x_Z$).

Notice also that, as stated in Proposition 19, the steady-state values of x vary across taxation levels such that the dynamic responses in Figure 3.5 depict the reversion to the steady state associated with each tax rate τ_A . Moreover, we shock directly the state variable x and keep a displacement of $\Delta = 1\%$ in all cases we consider. In other words, we take the *impact response* of the economy to a commodity price shock as given, and focus exclusively on how asset income taxation affects the *speed*

of reversion to the steady state due to the dynamics of the profit rate regardless of what causes its initial displacement. Later in this section, we discuss instead how the tax on asset income affects the impact response of the profit rate x to a commodity price shock.

Figure 3.5 contains three results. 1) The transitional dynamics are unambiguously slower in the “entry and quality regime,” i.e. for $x > x_Z$, compared to the “entry only regime,” i.e. for $x_N < x \leq x_Z$. This result holds irrespective of the specific value taken by the tax rate on asset income. Even in the benchmark case with no distortionary taxation, i.e. $\tau_A = 0$, the difference in the speed of reversion to the steady state across regimes is quite striking. In the “entry only regime,” the gap from the steady state is virtually closed 30 years after the shock. In the “entry and quality regime” instead, it takes more than 50 years to close the same initial gap of 1 percent. Importantly, the latter observations implicitly suggest that the persistence of the effects of unexpected commodity price changes greatly varies across growth regimes. 2) In each regime, the speed of reversion is decreasing in the tax rate τ_A . This is not surprising given the content of Lemma 20, which holds for all values that the asset income tax rate takes on the possible range, i.e., $\tau_A \in [0, 1)$. 3) Asset income taxation has larger effects in the “entry and quality regime” than in the “entry only regime.” This latter result and the different persistence of shocks across the two regimes highlight a distinctive and appealing feature of our theoretical structure. The dynamics of the model and so the interaction of asset income taxation with commodity prices have quantitatively different effects at different stages of economic development.

Up to now, we studied the speed of reversion to the steady state taking as given the initial displacement in x . However, asset income taxation also affects the impact response of the profit rate x to a commodity price shock. As formally stated in Proposition 21, the change in manufacturing spending $Y(p)$ induced by a change in

the commodity price p is increasing in the tax rate τ_A . This implies that for a given commodity price shock, the impact response of $x = Y(p)/\epsilon N(p)$ is larger at higher levels of asset income taxation. Hence, we next show that the initial displacement Δ induced by a commodity price change—*impact response*—is increasing in the tax rate on asset income τ_A . To understand why this is the case, let's consider a scenario in which there is a permanent fall in the commodity price, i.e., $p' < p$, and the economy operates under “global substitution,” such that $\Delta_Y \equiv Y(p') - Y(p) > 0$ for all $p' < p$ (see Proposition 16 for more details on global substitutability). Recall that the “long-run commodity price super-neutrality” result stated in Proposition 13 implies that $x^*(p') = x^*(p)$. Moreover, Proposition 21 states that $\Delta_Y(\tau'_A) > \Delta_Y(\tau_A)$ for all $\tau'_A > \tau_A$. With these results in mind, let's write the initial displacement Δ as

$$\Delta \equiv \frac{x_0}{x^*} - 1 = \frac{Y(p')/\epsilon N(p)}{Y(p')/\epsilon N(p')} - 1 = \frac{N(p')}{N(p)} - 1 \equiv \Delta_N > 0. \quad (3.35)$$

The combined of Propositions 13, 16, and 21 then implies that $\Delta_N(\tau'_A) > \Delta_N(\tau_A)$ for all $\tau'_A > \tau_A$. Therefore, the impact response Δ to a commodity price shock is increasing in the asset income tax τ_A .

To summarize, after an unexpected fall in the commodity price— p to p' —output in the manufacturing sector $Y(p)$ spikes to the new steady-state level $Y(p')$. After the initial spike Δ , the reversion to the new steady state $x^* = Y(p')/\epsilon N(p')$ is governed by positive net entry of firms— $N(p)$ to $N(p')$ with $N(p') > N(p)$. The magnitude and duration of each phase is affected by the level of asset income taxation.

Overall, the model suggests that taxation of asset income affects the entire response to a commodity price shock, i.e., *impact response* and *steady-state reversion*. Specifically, higher levels of asset income taxation imply a larger response on impact with a slower reversion to the steady state. In this latter respect, the analysis points to the *interaction* of fiscal policy with conditions in the commodity market

as an important mechanism through which external shocks to commodity prices are amplified and transmitted through the economy.

3.6.3 Policy Response to a Commodity Price Decline

In Section 3.6.2 above, we discussed the dynamic properties of the model in response to a commodity price permanent change keeping the level of asset income taxation fixed to a specified level. In this section instead, we discuss the scenario in which the government changes the tax rate on asset income in response to a decline in commodity prices.

Using equations (3.34) and (3.6), and the free-entry condition $A = NV = \beta WY$, we rewrite government revenues $R = \tau_A rA + \tau_\Omega p\Omega + \tau_C(Y_H + Y_F)$ as,

$$R(p) = \left[\frac{\rho\beta\tau_A}{1 - \tau_A} + \tau_C \right] Y(p) + \tau_\Omega p\Omega + \tau_C p(\Omega - O). \quad (3.36)$$

Lets consider the case in which $(\Omega - O) > 0$, i.e., the economy is a net commodity exporter. We believe this is the most relevant case for the current policy debate on commodity-rich countries. Equation (3.36) identifies three channels through which commodity prices affect government revenues: 1) the indirect effect through manufacturing expenditure $Y(p)$; 2) the direct *windfall effect* through taxation of commodity income $p\Omega$; and 3) the direct *expenditure effect*, which through the balance trade condition $Y_H + Y_F + p(O - \Omega) = Y$, manifests itself as taxation of exports $p(\Omega - O)$. Notice that the sign of the first effect depends on how manufacturing expenditure $Y(p)$ responds to commodity price changes. Specifically, a commodity price decline has a positive effect if the economy operates under “global substitution,” a negative effect under “global complementarity,” and it is neutral in a “Cobb-Douglas-like economy.”¹⁴ The sign of the second and third effect is instead unambiguous.

¹⁴ See Proposition 16 for more details on the properties of $Y(p)$.

At this stage, the notion of *fiscal dependence* is operational: if commodity-linked revenues, $\tau_\Omega p\Omega$, represent a large fraction of total fiscal revenues R , then government revenues are vulnerable to commodity price movements.

Lets consider now a scenario in which there is a permanent fall in the commodity price, i.e., $p' < p$, and government revenues fall such that $R(p') < R(p)$. If the government let the lump-sum rebate to the household decrease accordingly then all the dynamics would be those described in Section 3.6.2. However, if the government raises the tax rate on asset income to $\tau'_A > \tau_A$ in response to the tax revenue shortfall then the dynamic response to the shock change along two dimensions: 1) according to the discussion of Section 3.6.2, the shock to the commodity price is further amplified and its effects made more persistent; and 2) according to Proposition 19, the steady-state growth rate of TFP decreases. Thus, the commodity price change has an *indirect* adverse effect on long-run growth. This happens exclusively because of the government's reaction to the tax revenue shortfall. Overall, the analysis suggests that the short- and long-run performance of commodity-rich economies depends on the policy response implemented in the aftermath of the commodity price decline.

3.7 Conclusions

We develop a Schumpeterian small-open-economy model of endogenous growth. We focus on three policy relevant questions for commodity-exporting countries. How does the economy respond to external shocks to commodity prices? How is the dynamic response to commodity price changes affected by the structure of the tax code in place? And, how should governments adjust taxation in response to declining commodity income? The model is analytically transparent in that we derive closed-form solutions for the transitional dynamics. This allows us to compute welfare and disentangle the short- and long-run effects of distortionary taxation.

The results can be summarized as follows. 1) “*Long-run commodity price super-*

neutrality.” Commodity price changes affect the transitional dynamics of the model but have no effect on the steady-state growth rate of the economy. This is an important results since it suggests that, absent fiscal considerations, the long-run growth performance of an economy like ours is completely insulated from the conditions in the commodity market. 2) An increase in the tax on asset income has a positive level effect on manufacturing expenditure but it has an adverse effect on the steady-state growth rate of the economy. This implies if the government endogenously raises the tax rate on asset income in response to a shortfall of resource revenues then commodity price changes can still have indirect adverse effects on the steady-state growth rate of the economy. Notice that these negative long-run effects are exclusively the result of the (misguided) government’s response to changes in the economic environment. Finally, 3) a positive tax rate on asset income *amplifies* external shocks to the commodity price and *slows down* the reversion to the steady state after a commodity price shock. Overall, the theoratical analysis suggests that, in the aftermath of commodity price declines, the short- and long-run economic performance of a country is sensitive to the structure of the tax code in place and to the policy response implemented. In this sense, countries are *fiscally vulnerable*.

3.8 Tables and Figures

This section contains the main tables and figures of the essay.

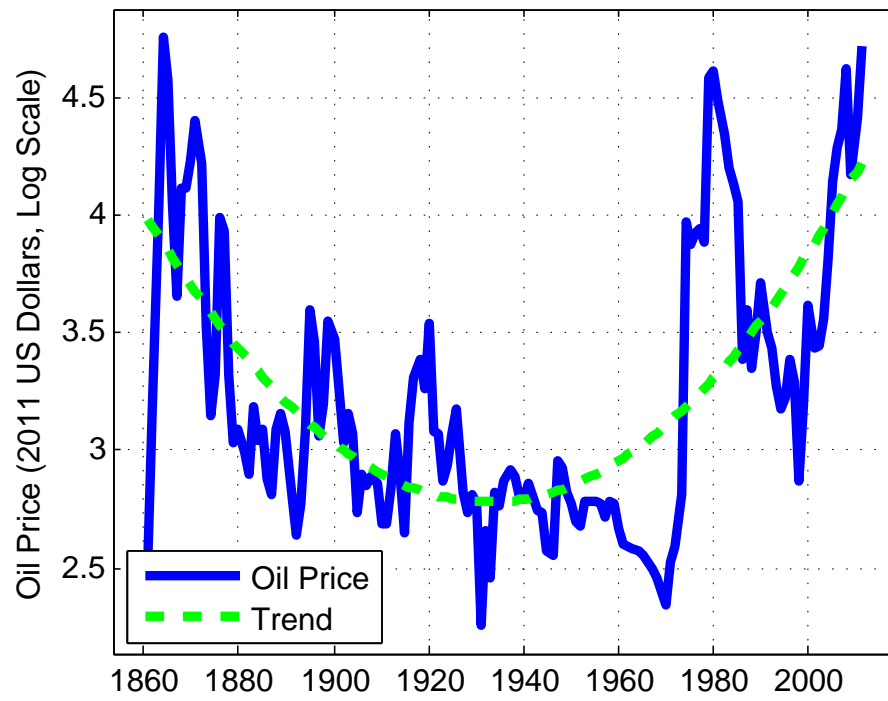


FIGURE 3.1: Real Oil Price and Trend, 1861-2011

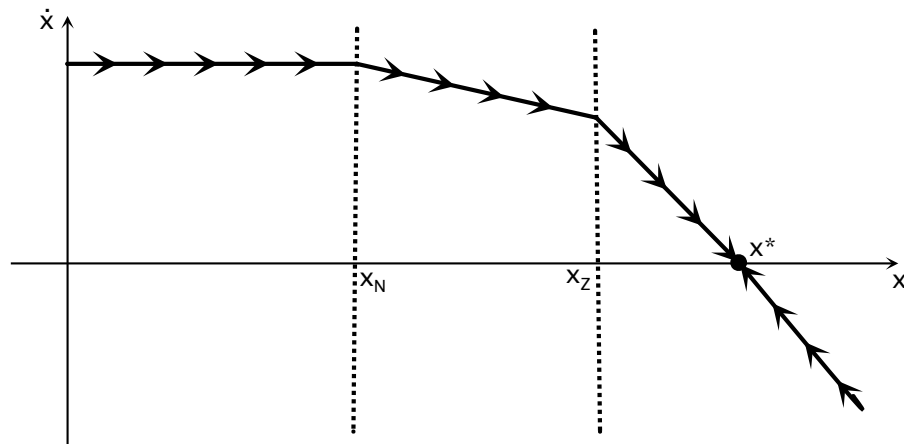


FIGURE 3.2: Global Equilibrium Dynamics

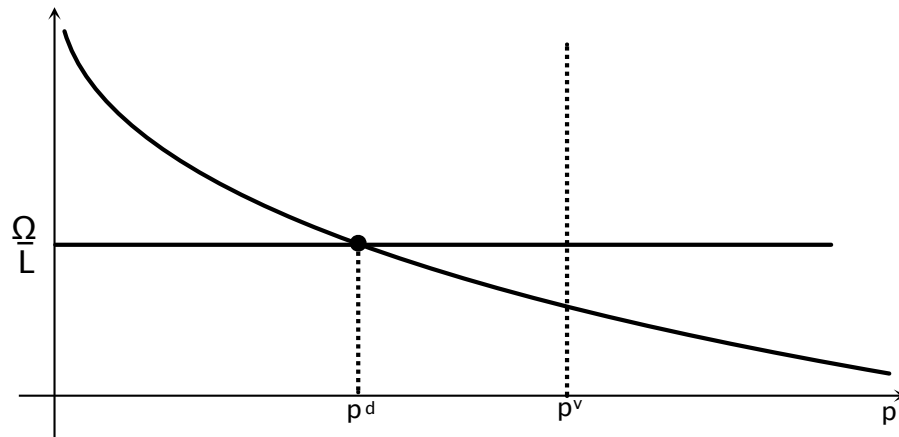


FIGURE 3.3: Dependence and Vulnerability

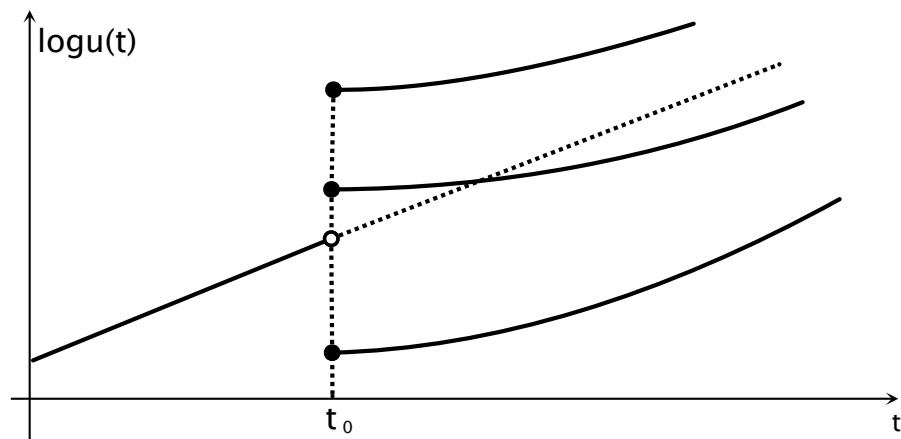


FIGURE 3.4: The Transition Path of Utility after a Commodity Price Boom in a Vulnerable Exporting Economy

Table 3.1: Baseline Parameters

Parameter	Interpretation	Value
$\epsilon/(\epsilon - 1)$	Mfg price markup	1.3
θ	Mfg prod. function: $X_i = Z_i^\theta F(L_{X_i} - \phi, M_i)$	0.15
ρ	Discount rate	0.02
δ	Death rate	0.035
β	Mfg entry cost: $V_i = \beta \cdot \frac{wY}{N}$	1

Table 3.2: Capital Income Tax Rates by Mendoza et al. (1994)

Year	Canada	France	Germany	Italy	Japan	U.K.	U.S.
1980	37.60	27.31	32.11	20.02	35.98	64.32	46.88
1996	50.66	26.11	23.91	33.86	42.61	47.17	39.62

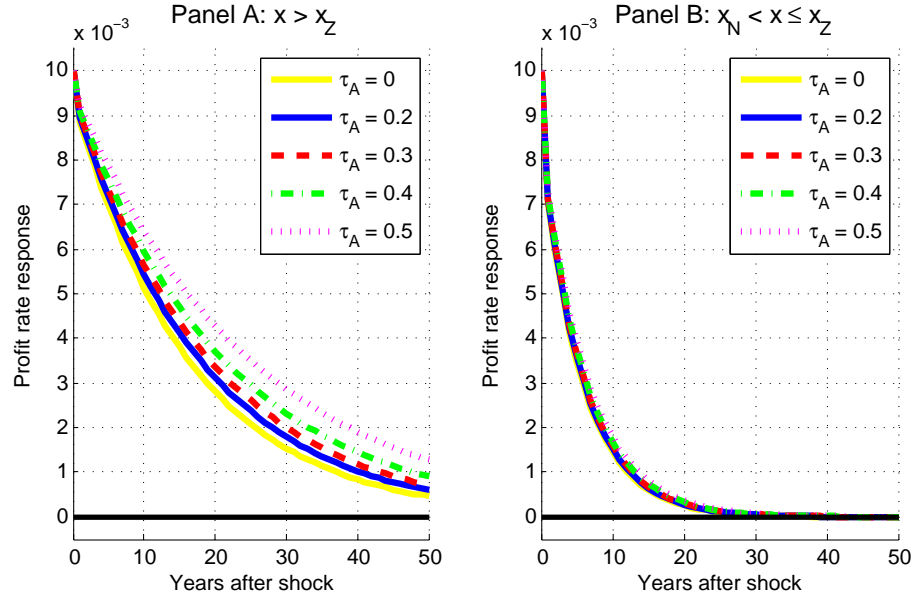


FIGURE 3.5: Dynamic Response to a “Profit Rate Shock”

Appendix A

Appendix to Chapter 1

A.1 Data Sources

The seasonally-adjusted monthly U.S. unemployment rate, participation rate (fraction of the population in the labor force), unemployment rates by age, gender, education, employment (thousands of persons), unemployment (thousands of persons), and short-term unemployment level (thousands of persons, number of unemployed persons for less than five weeks) are constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). Survey home page <http://www.bls.gov/cps/>. Seasonally-adjusted monthly employment (thousands of persons) by major sectors is constructed by the BLS from the Current Employment Statistics (CES). Survey home page <http://www.bls.gov/ces/>. Following Yedid-Levi (2013), I classify natural resources and mining, construction, durable goods manufacturing, and a half of professional and business services as investment sector, and the rest as consumption sector. State-level employment rates are from the Local Area Unemployment Statistics (LAUS) survey of the BLS for the period 1976:Q1-2012:Q2. Survey home page <http://www.bls.gov/lau/>. CPS micro

data for 1976:M1-2013:M2 are downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. Seasonally-adjusted quarterly hours, hours per worker, and real output per worker in the nonfarm business sector is constructed by the BLS from the Labor and Productivity Costs (LPC) release. Release home page <http://www.bls.gov/lpc>. BLS definitions: Business sector output (BSO) is the annual-weighted index constructed by the BLS after excluding from gross domestic product (GDP) the following outputs: general government, nonprofit institutions, paid employees of private households, and the rental value of owner-occupied dwellings. Nonfarm business sector output (NBSO) excludes from the business sector the farm sector. The series are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Monthly job-finding and job separation rates are calculated based on Shimer (2012). Vacancies are the monthly composite Help-Wanted Index constructed by Barnichon (2010).

A.2 Tables and Figures

Table A.1: Asymmetry in Employment Rates by Age

	25-34	35-44	45-54	55+
$Skew(\tilde{e})$	-0.728*** (0.003)	-0.690*** (0.004)	-0.610*** (0.001)	-0.407*** (0.009)
$Skew(\Delta e)$	-1.033*** (0.001)	-0.866*** (0.001)	-0.775** (0.010)	-0.434* (0.075)

Notes: e is a logged quarterly average of the seasonally-adjusted monthly U.S. employment rate (fraction of the labor force working in a given month, one minus the unemployment rate) by age group. \tilde{e} is the HP-filtered counterpart of e with smoothing parameter 10^5 . Data are from the CPS survey of the BLS for the period 1948:Q1-2013:Q1. Survey home page <http://www.bls.gov/cps/>. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. P-values (one-sided test) in parenthesis. ***, **, * denote statistical significance respectively at 1%, 5% and 10% level.

Table A.2: Asymmetry in Employment Rates by Education

	HSD	HSG	SC	CGPC
$Skew(\tilde{e})$	-0.553** (0.032)	-0.675** (0.022)	-0.396* (0.073)	-0.140 (0.298)
$Skew(\Delta e)$	-1.582* (0.090)	-1.527* (0.076)	-1.537* (0.082)	-1.088* (0.065)

Notes: e is a logged quarterly average of the seasonally-adjusted monthly U.S. employment rate (fraction of the labor force working in a given month, one minus the unemployment rate) by education group. \tilde{e} is the HP-filtered counterpart of e with smoothing parameter 10^5 . HSD, HSG, SC and CGPC denote respectively High School Dropouts, High School Graduates, Some College and College Graduates and Post-College Degree Holders. Data are from the CPS survey of the BLS for the period 1992:Q1-2013:Q1. Survey home page <http://www.bls.gov/cps/>. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. P-values (one-sided test) in parenthesis. **, * denote statistical significance respectively at 5% and 10% level.

Table A.3: Asymmetry in Employment Rates By Gender

	Men	Women
$Skew(\tilde{e})$	-0.690*** (0.004)	-0.417** (0.012)
$Skew(\Delta e)$	-0.938*** (0.002)	-1.372*** (0.009)

Notes: e is a logged quarterly average of the seasonally-adjusted monthly U.S. employment rate (fraction of the labor force working in a given month, one minus the unemployment rate) by gender. \tilde{e} is the HP-filtered counterpart of e with smoothing parameter 10^5 . Data are from the CPS survey of the BLS for the period 1948:Q1-2012:Q2. Survey home page <http://www.bls.gov/cps/>. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. P-values (one-sided test) in parenthesis. ***, ** denote statistical significance respectively at 1% and 5% level.

Table A.4: Asymmetry in Employment by Sector

	Consumption	Investment
$Skew(\tilde{e})$	-0.271* (0.055)	-0.266* (0.075)
$Skew(\Delta \tilde{e})$	-0.820*** (0.003)	-0.530* (0.065)

Notes: e is a logged quarterly average of seasonally-adjusted monthly employment (thousands of persons) as deviations from the HP trend with smoothing parameter 10^5 . Data are from the CES survey of the BLS for the period 1948:Q1-2012:Q2. Survey home page <http://www.bls.gov/ces/>. Following Yedid-Levi (2013), I classify natural resources and mining, construction, durable goods manufacturing, and a half of professional and business services as investment sector, and the rest as consumption sector. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. P-values (one-sided test) in parenthesis. ***, * denote statistical significance respectively at 1% and 10% level.

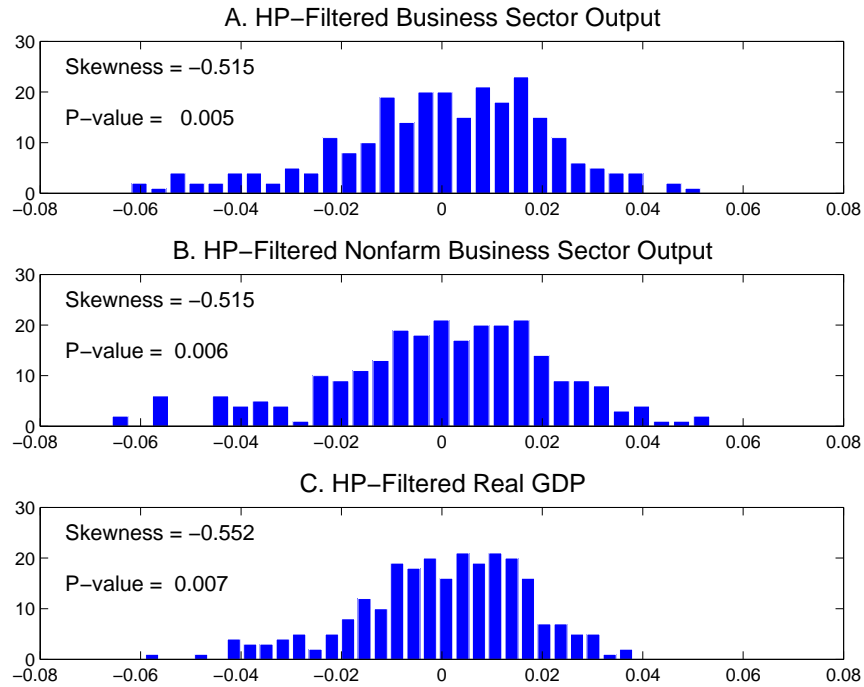


FIGURE A.1: Deepness Asymmetry in Quarterly U.S. Output, 1948:Q1-2012:Q2

Notes: All series are in logs as deviations from the HP trend with smoothing parameter 1600. In Panel A, business sector output (BSO) is the annual-weighted index constructed by the BLS after excluding from gross domestic product (GDP) the following outputs: general government, nonprofit institutions, paid employees of private households, and the rental value of owner-occupied dwellings. In Panel B, nonfarm business sector output (NBSO) excludes from the business sector the farm sector. In Panel C, real GDP is in billions of chained 2005 dollars. Data are seasonally-adjusted at the quarterly frequency for the period 1948:Q1-2012:Q2. BSO and NBSO data are from the LPC release of the BLS. Release home page <http://www.bls.gov/lpc>. BSO and NBSO data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Real GDP data are from the NIPA Table 1.1.6. and downloaded from the BEA website at <http://www.bea.gov/index.htm>.

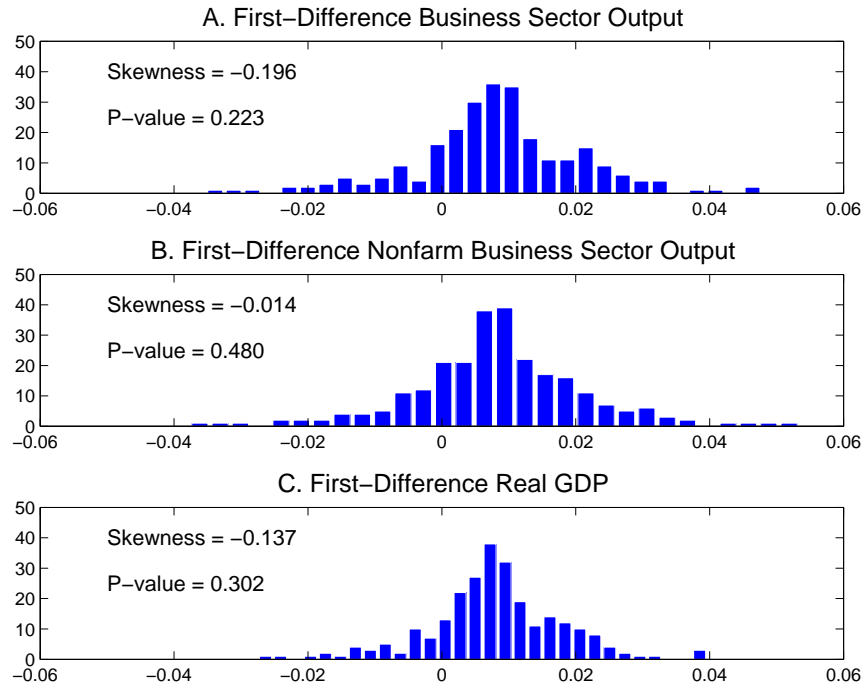


FIGURE A.2: Steepness Asymmetry in Quarterly U.S. Output, 1948:Q1-2012:Q2

Notes: All series are in log-first-differences. In Panel A, business sector output (BSO) is the annual-weighted index constructed by the BLS after excluding from gross domestic product (GDP) the following outputs: general government, nonprofit institutions, paid employees of private households, and the rental value of owner-occupied dwellings. In Panel B, nonfarm business sector output (NBSO) excludes from the business sector the farm sector. In Panel C, real GDP is in billions of chained 2005 dollars. Data are seasonally-adjusted at the quarterly frequency for the period 1948:Q1-2012:Q2. BSO and NBSO data are from the LPC release of the BLS. Release home page <http://www.bls.gov/lpc>. BSO and NBSO data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Real GDP data are from the NIPA Table 1.1.6. and downloaded from the BEA website at <http://www.bea.gov/index.htm>.



FIGURE A.3: Asymmetry in Employment Rates by State

Notes: In Panel A, red dot indicates the value of the skewness coefficient for the U.S. state-level employment rate (fraction of the labor force working in a given month, one minus the unemployment rate) as deviations from the HP trend with smoothing parameter 10^5 . In Panel B, red dot indicates the value of the skewness coefficient for the first-difference of the U.S. state-level employment rate. Employment rate is the logged quarterly average of the monthly series. On the x -axis, U.S. states are in alphabetical order. Data are from the LAUS survey of the BLS for the period 1976:Q1-2012:Q2. Survey home page <http://www.bls.gov/lau/>. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>.

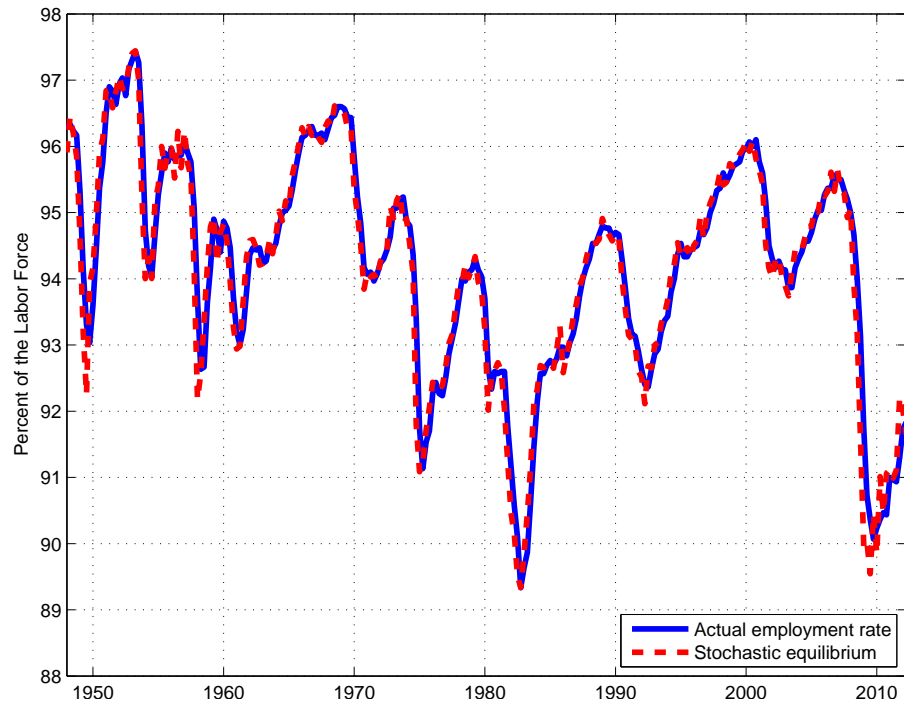


FIGURE A.4: Quarterly U.S. Employment Rate and Stochastic Equilibrium, 1948:Q1-2012:Q2

Notes: Solid line shows a quarterly average of the U.S. employment rate (fraction of the labor force working in a given month, one minus the unemployment rate) for the period 1948:Q1-2012:Q2. The seasonally-adjusted unemployment rate is from the CPS survey of the BLS for the period 1948:Q1-2012:Q2. Survey home page <http://www.bls.gov/cps/>. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Dashed line shows the counterfactual employment rate series under stochastic equilibrium. See Section 1.2.2 for details.

Table A.5: Calibration of the Standard DMP Model

Interpretation	Value	Source/Target
β <i>Discount factor</i>	0.9959	5% annual interest rate
δ <i>Separation rate</i>	0.036	JOLTS, 2001:M1-2011:M9
k <i>Vacancy cost</i>	0.1669	$\theta = 1$ when $z = 1$
α <i>Matching function: $m(v, u) = \mu v^\alpha u^{1-\alpha}$</i>	0.4	Brügemann (2008)
μ <i>Matching function scale</i>	0.607	Median unemployment rate of 5.6%
η <i>Worker Nash-bargaining weight</i>	0.6	Hosios (1990)'s condition
λ <i>Flow value of unemployment</i>	0.7221	0.73% of mean wage rate
ρ_z <i>Autocorrelation of exogenous state</i>	0.9642	LPC, 1948:Q1-2011:Q3
σ_ϵ <i>Standard deviation of shocks</i>	0.0055	LPC, 1948:Q1-2011:Q3

Notes: The separation rate, δ , is the monthly average of seasonally-adjusted total separations to employment in the nonfarm business sector. Data are from the JOLTS survey of the BLS for the period 2001:M1-2011:M9. Survey home page <http://www.bls.gov/jlt/>. To calibrate the stochastic process for labor productivity, I estimate an AR(1) process for the HP-filtered seasonally-adjusted quarterly real output per worker in the nonfarm business sector constructed by the BLS from the LPC release, for 1948:Q1-2011:Q3: $\ln(z_{t+1}) = \rho_z \ln(z_t) + \sigma_\epsilon \epsilon_{t+1}$ with $\epsilon_t \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. The HP-filter smoothing parameter is 10^5 . With quarterly data, we obtain an autocorrelation of $\hat{\rho}_z = 0.8963$ and a residual standard deviation of $\hat{\sigma}_\epsilon = 0.0091$, which yields $\hat{\rho}_z = 0.8963^{1/3} = 0.9642$ and $\hat{\sigma}_\epsilon = 0.0055$ at monthly frequency. Following Tauchen (1986), I approximate the continuous-valued AR(1) process for $\ln(z_t)$ through a \mathcal{S} -state Markov chain, having a discrete state space $\{z_1, \dots, z_{\mathcal{S}}\}$ and transition probabilities $\pi_{s,s'} = \Pr\{z_{t+1} = z_{s'} | z_t = z_s\}$. I set the number of grid points for the state space to $\mathcal{S} = 9$. Release home page <http://www.bls.gov/lpc>. Nonfarm business sector output (NBSO) excludes from the business sector the farm sector. Business sector output (BSO) is the annual-weighted index constructed by the BLS after excluding from gross domestic product (GDP) the following outputs: general government, nonprofit institutions, paid employees of private households, and the rental value of owner-occupied dwellings. Data are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>.

Table A.6: Volatility and Comovement in U.S. Data and DMP Model

	Data	DMP Model $\lambda = 0.73 \cdot \bar{\omega}$	DMP Model $\lambda = 0.95 \cdot \bar{\omega}$
A. Volatility			
$sd(u)/sd(p)$	9.851	1.303	7.864
$sd(\phi)/sd(p)$	7.213	1.401	8.599
$sd(v)/sd(p)$	8.131	2.284	14.167
B. Comovement			
$corr(u, v)$	-0.858	-0.900	-0.880
$corr(u, p)$	-0.412	-0.958	-0.934
$corr(\phi, p)$	0.383	0.999	0.974
$corr(v, p)$	0.433	0.986	0.954

Notes: The seasonally-adjusted unemployment rate, u , is from the CPS survey of the BLS. Survey home page <http://www.bls.gov/cps/>. Labor productivity, p , is seasonally-adjusted quarterly real output per worker in the nonfarm business sector constructed by the BLS from the LPC release. Release home page <http://www.bls.gov/lpc>. Nonfarm business sector output (NBSO) excludes from the business sector the farm sector. Business sector output (BSO) is the annual-weighted index constructed by the BLS after excluding from gross domestic product (GDP) the following outputs: general government, nonprofit institutions, paid employees of private households, and the rental value of owner-occupied dwellings. The series are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Job-finding rates, ϕ , are calculated based on Shimer (2012). Vacancies, v , are the composite Help-Wanted Index constructed by Barnichon (2010). The variables u , ϕ , and v are quarterly averages of monthly series. All series cover the period 1948:Q1-2011:Q3. DMP model refers to the standard DMP model with Nash-Bargaining and a constant exogenous rate of job destruction. λ and $\bar{\omega}$ denote respectively the flow value of unemployment and the mean wage rate. Model simulated data are quarterly averages of 765 observations at the monthly frequency. The statistics reported are averages across 500 replications. All variables are reported in logs as deviations from the HP trend with smoothing parameter 10^5 .

Table A.7: Asymmetry in U.S. Data and DMP Model

	Data	DMP Model $\lambda = 0.73 \cdot \bar{\omega}$	DMP Model $\lambda = 0.95 \cdot \bar{\omega}$
A. Skewness in levels			
$Skew(e)$	-0.591***	-0.105	-0.898
$Skew(y)$	-0.345**	-0.000	-0.267
$Skew(\phi)$	-0.249**	-0.068	-0.649
$Skew(v)$	-0.469***	-0.081	-0.760
B. Skewness in growth rates			
$Skew(\Delta e)$	-0.958***	0.018	0.158
$Skew(\Delta y)$	-0.170	0.002	0.089
$Skew(\Delta \phi)$	0.235***	0.000	0.007
$Skew(\Delta v)$	-0.613***	0.012	0.110

Notes: Employment rate, e , is the fraction of the labor force working in a given month, one minus the unemployment rate. The seasonally-adjusted unemployment rate is from the CPS survey of the BLS. Survey home page <http://www.bls.gov/cps/>. Output, y , is industrial production (IP). The series are downloaded from the FRED website at <http://research.stlouisfed.org/fred2/>. Job-finding rates, ϕ , are calculated based on Shimer (2012). Vacancies, v , are the composite Help-Wanted Index constructed by Barnichon (2010). The variables e , y , ϕ , and v are quarterly averages of monthly series. All series cover the period for period 1948:Q1-2011:Q3. DMP model refers to the standard DMP model with Nash-Bargaining and a constant exogenous rate of job destruction. λ and $\bar{\omega}$ denote respectively the flow value of unemployment and the mean wage rate. Model simulated data are quarterly averages of 765 observations at the monthly frequency. The statistics reported are averages across 500 replications. In Panel A, all variables are reported in logs as deviations from the HP trend with smoothing parameter 10^5 . In Panel B, all variables are reported as 3-months log-differences. ***, ** denote statistical significance respectively at 1% and 5% level.

Table A.8: Volatility Shares by Age/Education Group

	16-24	25-34	35-44	45-54	55-64	65+
A. Cov. not incl.						
HSD	17.90	2.56	1.03	1.31	0.92	0.04
HSG/SC	26.57	22.27	7.44	10.93	4.18	0.25
CGPC	0.16	0.85	0.94	1.49	1.08	0.08
B. Cov. incl.						
HSD	9.43	5.18	3.79	4.38	2.96	0.42
HSG/SC	19.87	19.15	7.70	8.66	5.98	1.03
CGPC	1.55	3.80	2.04	1.65	1.92	0.50

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. HSD, HSG, SC and CGPC denote respectively High School Dropouts, High School Graduates, Some College and College Graduates and Post-College Degree Holders. Seasonal adjustment is implemented with a 13-term symmetric moving average. The statistics reported are percentage shares of total U.S. unemployment rate variance attributed to each age-education group. “Cov. not incl.” means covariance terms are ignored such that total variation is the sum of the variables’ variances. “Cov. incl.” means total variation includes covariance terms such that total variation is the sum of the variables’ variances plus two times their covariance.

Table A.9: Unemployment versus Participation Margin by Age/Education Group

	16-24	25-34	35-44	45-54	55-64	65+
A. Unemployment margin						
HSD	93.36	97.41	98.23	97.72	98.92	93.75
HSG/SC	98.76	99.71	99.70	99.66	99.67	97.51
CGPC	95.27	99.56	99.63	99.26	98.18	97.21
B. Participation margin						
HSD	6.64	2.59	1.77	2.28	1.08	6.25
HSG/SC	1.24	0.29	0.30	0.34	0.33	2.49
CGPC	4.73	0.44	0.37	0.74	1.82	2.79

Notes: Data are from the CPS micro files for 1976:M1-2013:M2 and downloaded from the NBER website at http://www.nber.org/data/cps_basic.html. HSD, HSG, SC and CGPC denote respectively High School Dropouts, High School Graduates, Some College and College Graduates and Post-College Degree Holders. Seasonal adjustment is implemented with a 13-term symmetric moving average. Data are logged and HP-filtered with smoothing parameter 129,000 at the monthly frequency. See Ravn and Uhlig (2002) for a thorough discussion on the choice of the HP smoothing parameter and the frequency of observations. The statistics reported are percentage shares of total unemployment shares variance attributed to the unemployment and participation margin by each education group. Covariance terms are not included such that total variation is the sum of the variables' variances.

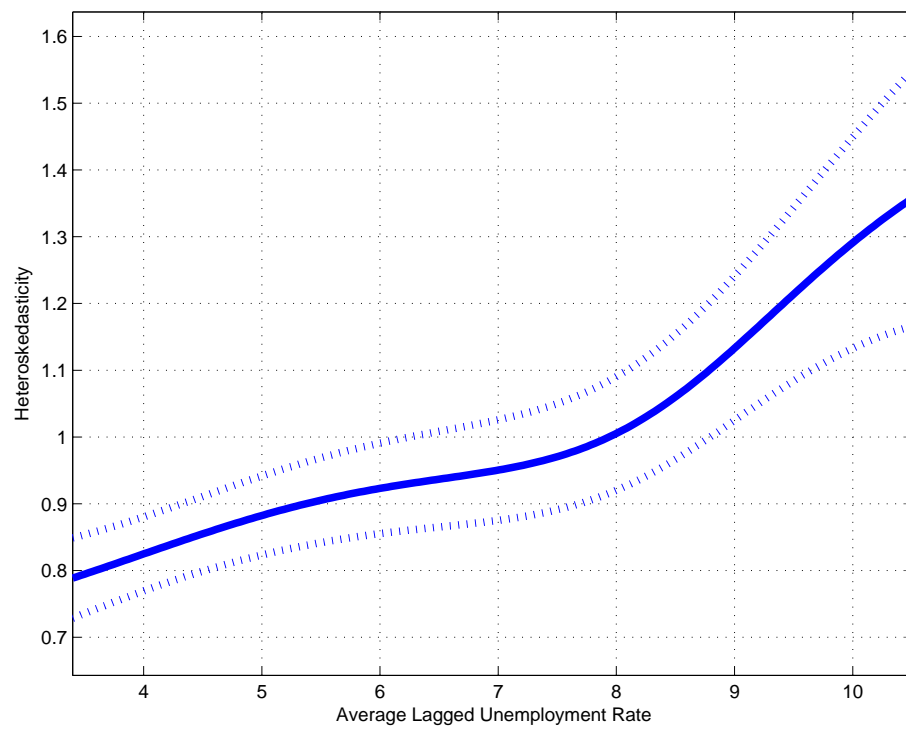


FIGURE A.5: Conditional Heteroskedasticity in Quarterly U.S. Unemployment Rate, 1959:Q1-2012:Q2

A.3 Proofs

Proof of Proposition 6. Standard reasoning provides the reservation productivity,

$$zx = \lambda + \frac{\eta k}{1 - \eta} \cdot \theta - \xi \beta (1 - \delta) \cdot \int_x^{x_M} S(y) dG(y). \quad (\text{A.1})$$

Rearrange equation (A.1) to get,

$$\theta = \frac{1 - \eta}{\eta k} \cdot \left[zx - \lambda + \xi \beta (1 - \delta) \cdot \int_x^{x_M} S(y) dG(y) \right]. \quad (\text{A.2})$$

In equation (A.2), as long as the term in square brackets is larger than zero, $\lim_{k \searrow 0} \theta(k) = +\infty$ and $\lim_{k \searrow 0} e(k) = 1$. On the other hand, as the term in square brackets falls below zero, $\lim_{k \searrow 0} \theta(k) = 0$ and $\lim_{k \searrow 0} e(k) = 0$. ■

Lemma 22 (Convexity of market tightness). *At any stochastic equilibrium consistent with a constant exogenous rate of job destruction, the tightness ratio, θ , is a strictly increasing and convex function of the exogenous state, z , i.e., $\theta = \varphi(z)$ with $\varphi'(\cdot) > 0$ and $\varphi''(\cdot) > 0$.*

Proof of Lemma 22. At any stochastic equilibrium consistent with a constant exogenous rate of job destruction, the total match surplus S_s is,

$$S_s = \frac{z_s - \lambda}{1 - \beta[1 - \delta - \eta\phi(\theta_s)]}$$

for $s = 1, \dots, \mathcal{S}$. After substituting S_s into the free-entry condition, $k = \beta \rho(\theta_s)(1 - \eta) S_s$, using $\phi(\theta) = \mu \theta^\alpha$, and rearranging terms,

$$\underbrace{k[1 - \beta(1 - \delta)]}_{\mathcal{A} > 0} = \underbrace{\beta \mu (1 - \eta)}_{\mathcal{B} > 0} (z - \lambda) \theta^{\alpha-1} - \underbrace{k \beta \mu \eta}_{\mathcal{C} > 0} \theta^\alpha,$$

where we omit the state subscript, s , for notational convenience. I can further rearrange the equation above to get θ as an implicit function $F(\theta, z)$ of z ,

$$F(\theta, z) \equiv \mathcal{A}\theta^{1-\alpha} + \mathcal{C}\theta - \mathcal{B}z = -\mathcal{B}\lambda.$$

By applying the implicit function theorem,

$$\frac{d\theta}{dz} = -\frac{\partial F(\theta, z)/\partial z}{\partial F(\theta, z)/\partial \theta} = \frac{\mathcal{B}}{\mathcal{A}(1-\alpha)\theta^{-\alpha} + \mathcal{C}} > 0,$$

implying that θ is strictly increasing in z . To further characterize the shape of the relationship between the market-tightness ratio and the exogenous state, I compute $d^2\theta/dz d\theta$,

$$\frac{d^2\theta}{dz d\theta} = \frac{\alpha(1-\alpha)\mathcal{A}\theta^{-(1+\alpha)}}{[\mathcal{A}(1-\alpha)\theta^{-\alpha} + \mathcal{C}]^2} > 0 \quad (\text{A.3})$$

such that θ is strictly convex in z , and this proves the lemma. ■

Proof of Proposition 8. Part (i). From Lemma 22, the tightness ratio is a strictly increasing and convex function of the exogenous state z , i.e., $\theta = \varphi(z)$ with $\varphi'(\cdot) > 0$ and $\varphi''(\cdot) > 0$. By Jensen's inequality,

$$\mathbb{E}[\theta] \equiv \mathbb{E}[\varphi(z)] > \varphi(\mathbb{E}[z]).$$

By Assumption 3, $\mathbb{E}[z] = z_{sm}$ such that $\varphi(\mathbb{E}[z]) = \varphi(z_{sm})$ and $\mathbb{E}[\varphi(z)] > \varphi(z_{sm})$. Hence,

$$\Delta\theta_{MMD} \equiv \mathbb{E}[\theta] - \theta_{sm} > 0$$

and this proves part (i) of the proposition. **Part (ii).** At any stochastic equilibrium consistent with a constant exogenous rate of job destruction, the employment

rate, job-finding rate, vacancies, and log output are strictly increasing and concave functions of the tightness ratio θ . Consider a generic increasing and concave function $\xi(\theta)$, with $\xi'(\cdot) > 0$ and $\xi''(\cdot) < 0$. By Jensen's inequality, we know that $\mathbb{E}[\xi(\theta)] < \xi(\mathbb{E}[\theta])$, and by adding and subtracting $\xi(\theta_{sm})$ on the right hand side of the inequality, we get

$$\Delta\xi_{MMD} \equiv \mathbb{E}[\xi(\theta)] - \xi(\theta_{sm}) < \xi(\mathbb{E}[\theta]) - \xi(\theta_{sm}) < \xi'(\theta_{sm})\Delta\theta_{MMD} > 0.$$

The second term on the right hand side, $\xi(\mathbb{E}[\theta]) - \xi(\theta_{sm})$, is larger than zero given that $\Delta\theta_{MMD} \equiv \mathbb{E}[\theta] - \theta_{sm} > 0$ and $\xi'(\cdot) > 0$. The second inequality on the right hand side is a fundamental property of concave functions. Finally, the third term on the right hand side, $\xi'(\theta_{sm})\Delta\theta_{MMD}$, is larger than zero given $\xi'(\cdot) > 0$ and $\Delta\theta_{MMD} > 0$ from part (i), and this completes the proof of the proposition. ■

Proof of Corollary 9. The symmetry and uni-modality of the stationary distribution of the exogenous state, z , (Assumption 3), and the linear support of the distribution of the tightness ratio θ ,

$$\theta_s = \theta_{sm} + \frac{(s - s_m)}{(\mathcal{S} - s_m)} \cdot \Delta,$$

imply that $\Delta\theta_{MMD} \equiv \mathbb{E}[\theta] - \theta_{sm} = 0$. From Proposition 8, it follows that for any increasing and concave function of the tightness ratio $\xi(\theta)$ (i.e., employment rate, job-finding rate, vacancies, log output), $\Delta\xi_{MMD} \equiv \mathbb{E}[\xi(\theta)] - \xi(\theta_{sm}) < 0$, and this proves the corollary. ■

Proof of Proposition 10. Part (i). The state space of the tightness ratio θ is the finite set $\Theta = \{\theta_1, \dots, \theta_{\mathcal{S}}\}$. In order to isolate the effects of an increase in volatility, let's assume that $\theta_s \in \Theta$ takes on values, $\theta_1 < \dots < \theta_{sm} < \dots < \theta_{\mathcal{S}}$, that are symmetrically spaced around the median θ_{sm} , satisfying the condition:

$$\theta_s = \theta_{s_m} + \frac{(s - s_m)}{(\mathcal{S} - s_m)} \cdot \Delta$$

for $s \in \{1, \dots, \mathcal{S}\}$ and $\Delta > 0$. The parameter Δ controls the range of variation of the support of the distribution of θ . Notice that by construction, changes in Δ have no effect on the median value θ_{s_m} . Given the symmetry assumption $\pi_\infty^s = \pi_\infty^{\mathcal{S}-s+1}$ for $s = 1, \dots, s_m$ (Assumption 3), also the expected value of θ under the stationary distribution π_∞ is invariant to changes in Δ and equal to the median value θ_{s_m} , i.e., $\mathbb{E}[\theta] = \sum_{s=1}^{\mathcal{S}} \pi_\infty^s \theta_s = \theta_{s_m}$ for all $\Delta \geq 0$. Hence, the parameter Δ acts like a mean-median-preserving spread in the distribution of θ .

In any equilibrium of the model consistent with a constant exogenous rate of job destruction δ , the unemployment rate follows a \mathcal{S} -state Markov chain with stochastic equilibrium

$$u_s = \frac{\delta}{\delta + \phi(\theta_s)} \quad (\text{A.4})$$

for $s \in \{1, \dots, \mathcal{S}\}$. The function $f(\theta) = \frac{\delta}{\delta + \phi(\theta)}$ is differentiable, decreasing, $f'(\cdot) < 0$, and convex, $f''(\cdot) > 0$, with $f(0) = 1$ and $\lim_{\theta \rightarrow \infty} f(\theta) = 0$.

Let $\Delta u_{MMD} \equiv \mathbb{E}[u] - u_{s_m}$ denote the difference between the expected and the median value of unemployment, and use $u = f(\theta)$ to write Δu_{MMD} as:

$$\begin{aligned} \Delta u_{MMD} &= \sum_{s=1}^{\mathcal{S}} \pi_\infty^s f(\theta_s) - f(\theta_{s_m}) = (\pi_\infty^{s_m} - 1) f(\theta_{s_m}) + \pi_\infty^1 f(\theta_{s_m} - \Delta) + \dots \\ &\dots + \pi_\infty^{s_m-1} f\left(\theta_{s_m} - \frac{\Delta}{\mathcal{S}-s_m}\right) + \dots + \pi_\infty^{s_m+1} f\left(\theta_{s_m} + \frac{\Delta}{\mathcal{S}-s_m}\right) + \dots + \pi_\infty^{\mathcal{S}} f(\theta_{s_m} + \Delta). \end{aligned}$$

By using the symmetry assumption $\pi_\infty^s = \pi_\infty^{\mathcal{S}-s+1}$ for $s = 1, \dots, s_m$ (Assumption 3), we can rewrite the expression above as,

$$\begin{aligned}\Delta u_{MMD} &= (\pi_{\infty}^{s_m} - 1) f(\theta_{s_m}) + \pi_{\infty}^1 [f(\theta_{s_m} - \Delta) + f(\theta_{s_m} + \Delta)] + \dots \\ &\quad \dots + \pi_{\infty}^{s_m-1} \left[f\left(\theta_{s_m} - \frac{\Delta}{\mathcal{S} - s_m}\right) + f\left(\theta_{s_m} + \frac{\Delta}{\mathcal{S} - s_m}\right) \right].\end{aligned}$$

We are interested in how Δu_{MMD} changes as Δ increases:

$$\begin{aligned}\frac{\partial \Delta u_{MMD}}{\partial \Delta} &= \pi_{\infty}^1 [f'(\theta_{s_m} + \Delta) - f'(\theta_{s_m} - \Delta)] + \dots \\ &\quad \dots + \frac{\pi_{\infty}^{s_m-1}}{\mathcal{S} - s_m} \left[f'\left(\theta_{s_m} + \frac{\Delta}{\mathcal{S} - s_m}\right) - f'\left(\theta_{s_m} - \frac{\Delta}{\mathcal{S} - s_m}\right) \right].\end{aligned}\quad (\text{A.5})$$

Using $f'(\cdot) < 0$, we rewrite (A.5) in a slightly different way,

$$\begin{aligned}\frac{\partial \Delta u_{MMD}}{\partial \Delta} &= \pi_{\infty}^1 [f'(\theta_{s_m} - \Delta) - f'(\theta_{s_m} + \Delta)] + \dots \\ &\quad \dots + \frac{\pi_{\infty}^{s_m-1}}{\mathcal{S} - s_m} \left[\left| f'\left(\theta_{s_m} - \frac{\Delta}{\mathcal{S} - s_m}\right) \right| - \left| f'\left(\theta_{s_m} + \frac{\Delta}{\mathcal{S} - s_m}\right) \right| \right].\end{aligned}\quad (\text{A.6})$$

Since $f'(\cdot) < 0$ and $f''(\cdot) > 0$, $f'(\theta_{s_m} - \epsilon) > f'(\theta_{s_m} + \epsilon)$ for all $\epsilon > 0$, and all terms in square brackets on the right hand side of (A.6) are strictly positive,

$$f'(\theta_{s_m} - \Delta) > f'(\theta_{s_m} + \Delta)$$

$$\left| f' \left(\theta_{s_m} - \frac{(s_m - 2) \Delta}{\mathcal{S} - s_m} \right) \right| > \left| f' \left(\theta_{s_m} + \frac{(s_m - 2) \Delta}{\mathcal{S} - s_m} \right) \right|$$

$$\begin{array}{ccc} \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \end{array}$$

$$\left| f' \left(\theta_{s_m} - \frac{\Delta}{\mathcal{S} - s_m} \right) \right| > \left| f' \left(\theta_{s_m} + \frac{\Delta}{\mathcal{S} - s_m} \right) \right|,$$

such that $\partial \Delta u_{MMD} / \partial \Delta > 0$ for all $\Delta > 0$. Given $e_s = 1 - u_s$ for $s \in \{1, \dots, \mathcal{S}\}$, $\Delta e_{MMD} = -\Delta u_{MMD}$. Hence, $\partial \Delta e_{MMD} / \partial \Delta = -\partial \Delta u_{MMD} / \partial \Delta < 0$, and this proves part (i) of the proposition. **Part (ii).** In any equilibrium of the model consistent with a constant exogenous rate of job destruction, the job-finding rate $\phi(\theta)$ follows a \mathcal{S} -state Markov chain with state space $\Phi = \{\phi(\theta_1), \dots, \phi(\theta_{\mathcal{S}})\}$. Given Assumption 1, the function $\phi(\theta)$ is differentiable, increasing, $\phi'(\cdot) > 0$, and concave, $\phi''(\cdot) < 0$.

Let $\Delta \phi_{MMD} \equiv \mathbb{E}[\phi(\theta)] - \phi(\theta_{s_m})$ denote the difference between the expected and the median value of the job-finding rate. Following the same steps as before for the unemployment rate, we rewrite $\Delta \phi_{MMD}$ as:

$$\begin{aligned} \Delta \phi_{MMD} &= (\pi_{\infty}^{s_m} - 1) \phi(\theta_{s_m}) + \pi_{\infty}^1 [\phi(\theta_{s_m} - \Delta) + \phi(\theta_{s_m} + \Delta)] + \dots \\ &\quad \dots + \pi_{\infty}^{s_m-1} \left[\phi \left(\theta_{s_m} - \frac{\Delta}{\mathcal{S} - s_m} \right) + \phi \left(\theta_{s_m} + \frac{\Delta}{\mathcal{S} - s_m} \right) \right]. \end{aligned}$$

We are interested in how $\Delta \phi_{MMD}$ changes as Δ increases:

$$\begin{aligned}
\frac{\partial \Delta \phi_{MMD}}{\partial \Delta} &= \pi_{\infty}^1 [\phi'(\theta_{s_m} + \Delta) - \phi'(\theta_{s_m} - \Delta)] + \dots \\
&\dots + \frac{\pi_{\infty}^{s_m-1}}{\mathcal{S} - s_m} \left[\phi' \left(\theta_{s_m} + \frac{\Delta}{\mathcal{S} - s_m} \right) - \phi' \left(\theta_{s_m} - \frac{\Delta}{\mathcal{S} - s_m} \right) \right]. \quad (\text{A.7})
\end{aligned}$$

Since $\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$, $\phi'(\theta_{s_m} + \epsilon) < \phi'(\theta_{s_m} - \epsilon)$ for all $\epsilon > 0$, such that all terms in square brackets on the right hand side of (A.7) are strictly negative. Hence, $\partial \Delta \phi_{MMD} / \partial \Delta < 0$ for all $\Delta > 0$, and this proves part (ii) of the proposition. **Part (iii).** In any equilibrium of the model consistent with a constant exogenous rate of job destruction, vacancies follow a \mathcal{S} -state Markov chain with stochastic equilibrium $v_s = \theta_s u_s$, for $s \in \{1, \dots, \mathcal{S}\}$. Let's write vacancies as $v_s = \theta_s f(\theta_s)$, where the function $f(\theta)$ is differentiable, decreasing, $f'(\cdot) < 0$, and convex, $f''(\cdot) > 0$.

Let $\Delta v_{MMD} \equiv \mathbb{E}[v] - v_{s_m}$ denote the difference between the expected and the median value of vacancies. By using the symmetry assumption $\pi_{\infty}^s = \pi_{\infty}^{\mathcal{S}-s+1}$ for $s = 1, \dots, s_m$ (Assumption 3), we can write Δv_{MMD} as,

$$\begin{aligned}
\Delta v_{MMD} &= (\pi_{\infty}^{s_m} - 1) \theta_{s_m} f(\theta_{s_m}) + \\
&+ \pi_{\infty}^1 [(\theta_{s_m} - \Delta) f(\theta_{s_m} - \Delta) + (\theta_{s_m} + \Delta) f(\theta_{s_m} + \Delta)] + \dots \\
&\dots + \pi_{\infty}^{s_m-1} \left[\left(\theta_{s_m} - \frac{\Delta}{\mathcal{S}-s_m} \right) f \left(\theta_{s_m} - \frac{\Delta}{\mathcal{S}-s_m} \right) + \left(\theta_{s_m} + \frac{\Delta}{\mathcal{S}-s_m} \right) f \left(\theta_{s_m} + \frac{\Delta}{\mathcal{S}-s_m} \right) \right].
\end{aligned}$$

We are interested in how Δv_{MMD} changes as Δ increases:

$$\begin{aligned}
\frac{\partial \Delta v_{MMD}}{\partial \Delta} &= \pi_{\infty}^1 [f(\theta_{s_m} + \Delta) - f(\theta_{s_m} - \Delta) + \\
&+ (\theta_{s_m} + \Delta) f'(\theta_{s_m} + \Delta) - (\theta_{s_m} - \Delta) f'(\theta_{s_m} - \Delta)] + \dots \\
&\dots + \frac{\pi_{\mathcal{C}}^{s_m-1}}{\mathcal{S}-s_m} \left[f\left(\theta_{s_m} + \frac{\Delta}{\mathcal{S}-s_m}\right) - f\left(\theta_{s_m} - \frac{\Delta}{\mathcal{S}-s_m}\right) + \right. \\
&\left. + \left(\theta_{s_m} + \frac{\Delta}{\mathcal{S}-s_m}\right) f'\left(\theta_{s_m} + \frac{\Delta}{\mathcal{S}-s_m}\right) - \left(\theta_{s_m} - \frac{\Delta}{\mathcal{S}-s_m}\right) f'\left(\theta_{s_m} - \frac{\Delta}{\mathcal{S}-s_m}\right) \right]. \quad (\text{A.8})
\end{aligned}$$

Notice that each term in square brackets on the right hand side of (A.8) takes the form,

$$\mathcal{E} \equiv f(\theta_{s_m} + \epsilon) - f(\theta_{s_m} - \epsilon) + (\theta_{s_m} + \epsilon) f'(\theta_{s_m} + \epsilon) - (\theta_{s_m} - \epsilon) f'(\theta_{s_m} - \epsilon).$$

By substituting the expressions for $f(\cdot)$ and $f'(\cdot)$ in \mathcal{E} , one can show that $\mathcal{E} < 0$ for all $\epsilon > 0$, such that all terms on the right hand side of (A.8) are strictly negative. Hence, $\partial \Delta v_{MMD} / \partial \Delta < 0$ for all $\Delta > 0$, and this proves part (iii) of the proposition.

Part (iv). In any equilibrium of the model consistent with a constant exogenous rate of job destruction, output follows a \mathcal{S} -state Markov chain with state space $\mathcal{Y} = \{y_1, \dots, y_S\}$. At the stochastic equilibrium, output is $y_s = z_s e_s$, where the employment rate e_s is a strictly increasing and concave function of the tightness ratio, $\xi(\theta)$ with $\xi'(\cdot) > 0$ and $\xi''(\cdot) < 0$. From Lemma 22, we know that in equilibrium the tightness ratio is strictly increasing and convex in the aggregate shock z , i.e., $\theta = \varphi(z)$ with $\varphi'(\cdot) > 0$ and $\varphi''(\cdot) > 0$. By inverting the function $\varphi(\cdot)$, it follows that $z = \phi(\theta)^{-1} \equiv \psi(\theta)$, with $\psi'(\cdot) > 0$ and $\psi''(\cdot) < 0$. Let's rewrite output as $y_s = \psi(\theta) \xi(\theta)$, and $\ln y_s = \ln \psi(\theta) + \ln \xi(\theta)$. It is easily established that log output

is strictly increasing and concave in the tightness ratio θ , as such following the same steps of parts (ii) and (iii) one can show that $\partial \Delta y_{MMD} / \partial \Delta < 0$ for all $\Delta > 0$, and this completes the proof of the proposition. ■

Proof of Proposition 11. Part (i). Consider a mean-median-preserving spread Δ in the distribution of the generic endogenous variable \tilde{x} with stochastic equilibrium,

$$\tilde{x}_s = \tilde{x}_{s_m} + \frac{(s - s_m)}{(\mathcal{S} - s_m)} \cdot \Delta \quad (\text{A.9})$$

for $s \in \{1, \dots, \mathcal{S}\}$ and $\Delta > 0$. An increase in the parameter Δ raises the variance of \tilde{x} , leaving unaltered both the mean and the median of the stochastic process. Let $\Delta \tilde{x}_{s,s'} \equiv (\tilde{x}_{s'} - \tilde{x}_s)$ denote the change in the variable \tilde{x} from the current state s to the next period state s' . From equation (A.9):

$$\Delta \tilde{x}_{s,s'} = \frac{(s' - s)}{(\mathcal{S} - s_m)} \cdot \Delta \quad (\text{A.10})$$

such that $\Delta \tilde{x}_{i,j} = -\Delta \tilde{x}_{j,i} > 0$, for all $j > i$, and $\Delta \tilde{x}_{s,s_m} = \Delta \tilde{x}_{\mathcal{S}-s+1,s_m}$, for all $s \in \{1, \dots, s_m - 1\}$. Since the model is stationary, $\mu_{\Delta \tilde{x}} \equiv \mathbb{E}[\Delta \tilde{x}] = 0$, and the skewness coefficient of $\Delta \tilde{x}$, $Skew[\Delta \tilde{x}] = \mathbb{E}[(\Delta \tilde{x} - \mu_{\Delta \tilde{x}})^3] / \sigma_{\Delta \tilde{x}}^3$, reduces to $Skew[\Delta \tilde{x}] = \mathbb{E}[\Delta \tilde{x}^3] / \sigma_{\Delta \tilde{x}}^3$. Because the sign of $Skew[\Delta \tilde{x}]$ is determined by the sign of the numerator, I focus without loss of generality on $NSkew[\Delta \tilde{x}] \equiv \mathbb{E}[\Delta \tilde{x}^3]$:

$$\begin{aligned} NSkew[\Delta \tilde{x}] &= \sum_{s=1}^{\mathcal{S}} \pi_{\infty}^s \sum_{s'=1}^{\mathcal{S}} \pi_{s,s'} \Delta \tilde{x}_{s,s'}^3 = \sum_{s=1}^{\mathcal{S}} \pi_{\infty}^s \left(\pi_{s,s_m} \Delta \tilde{x}_{s,s_m}^3 + \pi_{s_m,s} \Delta \tilde{x}_{s_m,s}^3 \right) + \\ &+ \sum_{s=1}^{s_m-1} \pi_{\infty}^s \left(\pi_{s,\mathcal{S}-s+1} \Delta \tilde{x}_{s,\mathcal{S}-s+1}^3 + \pi_{\mathcal{S}-s+1,s} \Delta \tilde{x}_{\mathcal{S}-s+1,s}^3 \right). \end{aligned} \quad (\text{A.11})$$

Using Assumption 3 on the symmetry of transition probabilities $\pi_{s,s'}$, and that by definition $\Delta\tilde{x}_{s,s} = 0$, for all $s \in \{1, \dots, \mathcal{S}\}$, I rewrite equation (A.11) as,

$$\begin{aligned}
NSkew[\Delta\tilde{x}] &= \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s,s_m} \left(\Delta\tilde{x}_{s,s_m}^3 + \Delta\tilde{x}_{\mathcal{S}-s+1,s_m}^3 \right) + \\
&+ \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s_m,s} \left(\Delta\tilde{x}_{s_m,s}^3 + \Delta\tilde{x}_{s_m,\mathcal{S}-s+1}^3 \right) + \quad (A.12) \\
&+ \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s,\mathcal{S}-s+1} \left(\Delta\tilde{x}_{s,\mathcal{S}-s+1}^3 + \Delta\tilde{x}_{\mathcal{S}-s+1,s}^3 \right).
\end{aligned}$$

Using equation (A.10) for the support of $\Delta\tilde{x}$, I rewrite equation (A.12) as,

$$\begin{aligned}
NSkew[\Delta\tilde{x}] &= \frac{\Delta^3}{(\mathcal{S} - s_m)^3} \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s,s_m} \left[\left(s_m - s \right)^3 + \left(s_m - \mathcal{S} + s - 1 \right)^3 \right] + \\
&+ \frac{\Delta^3}{(\mathcal{S} - s_m)^3} \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s_m,s} \left[\left(s - s_m \right)^3 + \left(\mathcal{S} - s + 1 - s_m \right)^3 \right] + (A.13) \\
&+ \frac{\Delta^3}{(\mathcal{S} - s_m)^3} \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s,\mathcal{S}-s+1} \left[\left(\mathcal{S} - s + 1 - s \right)^3 + \left(s - \mathcal{S} + s - 1 \right)^3 \right].
\end{aligned}$$

After substituting the expression for the median state, $s_m = (\mathcal{S} + 1)/2$, into equation (A.13), it is straightforward to check that all terms in square brackets on the right hand side equal zero, such that $NSkew[\Delta\tilde{x}] = 0$, for all $\Delta > 0$. Hence, regardless of the mean-median-preserving spread Δ , the unconditional distribution of $\Delta\tilde{x}_{s,s'}$ is symmetric, and this proves part (i) of the proposition. **Part (ii).** Assume the support of the distribution of \tilde{x} is left-skewed, such that $\Delta\tilde{x}_{s,s_m} > \Delta\tilde{x}_{\mathcal{S}-s+1,s_m}$, for $s \in \{1, \dots, s_m - 1\}$. Let's write again equation (A.12),

$$\begin{aligned}
NSkew[\Delta\tilde{x}] &= \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s,s_m} \left(\Delta\tilde{x}_{s,s_m}^3 + \Delta\tilde{x}_{\mathcal{S}-s+1,s_m}^3 \right) + \\
&+ \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s_m,s} \left(\Delta\tilde{x}_{s_m,s}^3 + \Delta\tilde{x}_{s_m,\mathcal{S}-s+1}^3 \right) + \\
&+ \sum_{s=1}^{s_m-1} \pi_{\infty}^s \pi_{s,\mathcal{S}-s+1} \left(\Delta\tilde{x}_{s,\mathcal{S}-s+1}^3 + \Delta\tilde{x}_{\mathcal{S}-s+1,s}^3 \right).
\end{aligned} \tag{A.14}$$

Since $\Delta\tilde{x}_{i,j} = -\Delta\tilde{x}_{j,i}$, for all $j > i$, the third term in square brackets on the right hand side of equation (A.14) equals zero. We can further rearrange equation (A.14),

$$NSkew[\Delta\tilde{x}] = \sum_{s=1}^{s_m-1} \pi_{\infty}^s \left(\pi_{s,s_m} - \pi_{s_m,s} \right) \left(\left| \Delta\tilde{x}_{s,s_m}^3 \right| - \left| \Delta\tilde{x}_{\mathcal{S}-s+1,s_m}^3 \right| \right) > 0. \tag{A.15}$$

In equation (A.15), the inequality $NSkew[\Delta\tilde{x}] > 0$, holds because $\pi_{s,s_m} > \pi_{s_m,s}$, for $s \in \{1, \dots, \mathcal{S}\}$, which is the condition that the stationary distribution of the Markov chain is uni-modal (Assumption 3), and $\Delta\tilde{x}_{s,s_m}^3 > \Delta\tilde{x}_{\mathcal{S}-s+1,s_m}^3$, for $s \in \{1, \dots, s_m - 1\}$, and this proves part (ii) of the proposition. **Part (iii).** Assume the support of the distribution of \tilde{x} is right-skewed, such that $\Delta\tilde{x}_{s,s_m} < \Delta\tilde{x}_{\mathcal{S}-s+1,s_m}$, for $s \in \{1, \dots, s_m - 1\}$. Following the same steps of part (ii),

$$NSkew[\Delta\tilde{x}] = \sum_{s=1}^{s_m-1} \pi_{\infty}^s \left(\pi_{s,s_m} - \pi_{s_m,s} \right) \left(\left| \Delta\tilde{x}_{s,s_m}^3 \right| - \left| \Delta\tilde{x}_{\mathcal{S}-s+1,s_m}^3 \right| \right) < 0, \tag{A.16}$$

and this completes the proof of the proposition. ■

A.4 Integrated Labor Market

This section details the building blocks of the integrated labor market version of the model presented in Section 3.2. I consider the labor market in steady state, i.e., $z_t = z_s$ for $s \in \{1, \dots, \mathcal{S}\}$ and all t . As in the segmented labor market version of the model, there are M types of workers index by $x \in \{x_1, \dots, x_M\}$. Total match surplus from being matched with a worker of type x if the economy is in state $s \in \{1, \dots, \mathcal{S}\}$ is

$$S_s(x) = \max \{S_s^c(x), 0\} \quad (\text{A.17})$$

with

$$S_s^c(x) = z_s x - \lambda + \beta [1 - \delta_s(x) - \eta \phi(\theta_s(\pi_s))] S_s(x). \quad (\text{A.18})$$

$S_s^c(x)$ is the value of continuing the match. The free-entry condition for employers is

$$k = \rho(\theta_s(\pi_s)) \beta (1 - \eta) \sum_x \pi_s(x) S_s(x), \quad (\text{A.19})$$

where the probability to come in contact with a worker of type x is $\pi_s(x) = u_s(x)/U_s$ where $U_s = \sum_x u_s(x)$ is aggregate unemployment.

Appendix B

Appendix to Chapter 2

B.1 Tables and Figures

Table B.1: Computers per Capita — Time Adoption Lags

Country	Lag	Ranking	Country	Lag	Ranking
Argentina	18.07275	(29)	Mauritania	20.84607	(59)
Australia	2.036133	(5)	Mauritius	16.53137	(26)
Austria	5.858276	(16)	Mexico	18.00513	(28)
Belgium	10.5116	(19)	Morocco	19.98181	(50)
Bolivia	20.0188	(51)	Namibia	18.30847	(30)
Botswana	19.42114	(40)	Netherlands	3.682251	(8)
Brazil	18.31384	(31)	New Zealand	4.631592	(13)
Canada	3.389893	(7)	Nicaragua	19.78955	(46)
Chile	16.39294	(25)	Norway	2.643188	(6)
Colombia	18.91455	(37)	Panama	19.50916	(43)
Costa Rica	13.38269	(21)	Papua New Guinea	18.67932	(35)
Denmark	1.885254	(4)	Paraguay	19.63745	(44)
Ecuador	19.73718	(45)	Peru	19.29553	(39)
Egypt	20.51685	(55)	Philippines	19.91785	(49)
El Salvador	19.86792	(48)	Portugal	15.91223	(24)
Finland	4.146606	(10)	Romania	18.40564	(34)
France	6.747559	(17)	Senegal	20.26416	(52)
Gabon	20.29541	(53)	Singapore	1.136475	(2)
Greece	17.95654	(27)	South Africa	18.32104	(32)
Guatemala	20.59973	(56)	Spain	13.43213	(22)
Honduras	20.68445	(57)	Sri Lanka	20.74939	(58)
Hong Kong	4.521729	(11)	Sweden	1.096558	(1)
Iceland	4.089233	(9)	Syria	20.32605	(54)
Indonesia	20.84692	(60)	Thailand	19.45422	(42)
Iran	18.37402	(33)	Togo	19.85657	(47)
Ireland	4.606201	(12)	Turkey	19.24939	(38)
Israel	10.05469	(18)	United Kingdom	4.941895	(14)
Italy	11.26416	(20)	Zimbabwe	18.83337	(36)
Japan	5.432739	(15)			
Jordan	19.43225	(41)			
Luxembourg	1.727661	(3)	Mean Lag	14.13115	
Malaysia	15.33167	(23)	Std. Dev. Lag	7.080635	

Notes: 60 countries. Personal computer invention year: 1973. Lag measures how many years ago the United States last had the usage level of computers per capita that each country had in the benchmark year 2002.

Table B.2: Internet Users per Capita — Time Adoption Lags

Country	Lag	Ranking	Country	Lag	Ranking
Algeria	10.28662	(53)	Malaysia	3.794067	(15)
Argentina	6.824463	(25)	Mauritius	6.882202	(26)
Australia	0.294189	(1)	Mexico	6.969482	(28)
Austria	2.390869	(10)	Morocco	8.960205	(47)
Belgium	3.636719	(14)	Namibia	8.857544	(45)
Bolivia	8.640015	(44)	Netherlands	0.894165	(5)
Botswana	8.585815	(43)	New Zealand	1.196045	(7)
Brazil	7.352173	(30)	Nicaragua	9.872314	(50)
Canada	0.857910	(4)	Norway	4.008545	(17)
Chile	4.796753	(20)	Pakistan	11.67236	(59)
Colombia	7.974365	(36)	Panama	7.759399	(35)
Costa Rica	5.601318	(23)	Papua New Guinea	10.58838	(54)
Denmark	0.780395	(2)	Paraguay	10.12036	(51)
Ecuador	8.244019	(39)	Peru	7.089111	(29)
El Salvador	7.982544	(37)	Philippines	8.312256	(40)
Finland	0.829468	(3)	Portugal	5.5448	(21)
France	4.065430	(18)	Romania	6.948608	(27)
Gabon	9.673218	(49)	Senegal	11.37109	(57)
Greece	6.384277	(24)	Singapore	1.021118	(6)
Guatemala	8.565063	(42)	South Africa	7.612793	(32)
Guinea-Bissau	10.95874	(56)	Spain	5.592651	(22)
Haiti	11.70679	(60)	Sri Lanka	11.41138	(58)
Honduras	8.911987	(46)	Switzerland	3.312378	(13)
India	10.27417	(52)	Thailand	7.414429	(31)
Indonesia	9.499390	(48)	Togo	8.479736	(41)
Ireland	4.322144	(19)	Turkey	7.729248	(33)
Israel	3.955688	(16)	United Kingdom	2.247925	(9)
Italy	3.278809	(12)	Zimbabwe	8.099609	(38)
Japan	1.834595	(8)			
Jordan	7.751953	(34)			
Kenya	10.91492	(55)			
Lesotho	11.76355	(61)	Mean Lag	6.651275	
Luxembourg	3.027222	(11)	Std. Dev. Lag	3.344070	

Notes: 61 countries. Personal computer invention year: 1983. Lag measures how many years ago the United States last had the usage level of internet users per capita that each country had in the benchmark year 2002.

Table B.3: Cell Phones per Capita — Time Adoption Lags

Country	Lag	Ranking	Country	Lag	Ranking
Algeria	13.23938	(51)	Mali	15.01709	(62)
Argentina	5.828003	(20)	Mauritania	7.829346	(27)
Bangladesh	14.12561	(57)	Mauritius	3.338989	(6)
Benin	11.00305	(43)	Mexico	4.016602	(9)
Bolivia	7.537231	(26)	Morocco	4.757568	(14)
Botswana	4.310547	(11)	Mozambique	13.10181	(50)
Brazil	5.260010	(17)	Namibia	8.349609	(29)
Burkina Faso	14.40405	(61)	Nicaragua	10.21289	(38)
Burundi	14.35815	(59)	Niger	17.04858	(66)
Cameroon	10.07715	(36)	Nigeria	13.06262	(49)
Canada	2.209717	(3)	Pakistan	14.22424	(58)
Central African Republic	15.94641	(64)	Panama	5.448120	(18)
Chad	15.34192	(63)	Papua New Guinea	15.98962	(65)
Chile	1.349121	(1)	Paraguay	3.467285	(7)
Colombia	7.289795	(24)	Peru	8.145386	(28)
Costa Rica	7.511719	(25)	Philippines	5.583862	(19)
Dominican Republic	4.999512	(15)	Romania	4.342285	(12)
Ecuador	7.180908	(23)	Rwanda	13.29028	(52)
Egypt	8.877563	(30)	Senegal	9.341309	(32)
El Salvador	6.382080	(21)	South Africa	3.169434	(5)
Equatorial Guinea	9.151611	(31)	Sri Lanka	9.728882	(35)
Ethiopia	17.66187	(68)	Syria	11.78503	(44)
Gabon	4.706177	(13)	Tanzania	12.26221	(46)
Ghana	12.11926	(45)	Thailand	3.948608	(8)
Guatemala	6.726196	(22)	Togo	10.87634	(41)
Guinea	13.59229	(55)	Turkey	2.515503	(4)
Haiti	12.73389	(47)	Uganda	12.82593	(48)
Honduras	9.642090	(34)	Uruguay	5.088867	(16)
India	13.32336	(53)	Zambia	13.34106	(54)
Indonesia	9.444824	(33)	Zimbabwe	10.73901	(40)
Iran	10.89673	(42)			
Jordan	4.210083	(10)			
Kenya	10.18225	(37)			
Lesotho	10.28149	(39)			
Madagascar	13.71362	(56)			
Malawi	14.39160	(60)	Mean Lag	9.449093	
Malaysia	2.157715	(2)	Std. Dev. Lag	4.387134	

Notes: 68 countries. Cell phones invention year: 1973. Lag measures how many years ago the United States last had the usage level of cell phones per capita that each country had in the benchmark year 2002.

Table B.4: Volatility and Time Adoption Lags — Computers per Capita

Independent Variable	Full Sample (60 countries, 1960-2002)			OECD Sample (24 countries, 1960-2002)		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Volatility</i>	163.712*** (24.855)		149.402*** (29.397)	301.506*** (81.845)		304.425*** (86.185)
<i>Growth</i>		-181.898*** (67.123)	-133.012** (55.706)		61.662 (208.090)	-31.825 (169.365)
R^2	0.298	0.121	0.360	0.324	0.004	0.325
Observations	60	60	60	24	24	24

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the usage level of computers per capita that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates over the sample period 1960-2002 for each country. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, ** indicate respectively statistical significance at the 1 and 5 percent level.

Table B.5: Volatility and Time Adoption Lags — Internet per Capita

Independent Variable	Full Sample (61 countries, 1960-2002)			OECD Sample (23 countries, 1960-2002)		
	(1)	(2)	(3)	(4)	(5)	(6)
<i>Volatility</i>	74.900*** (11.334)		65.190*** (13.590)	104.349*** (35.033)		101.149** (36.265)
<i>Growth</i>		-98.256*** (29.963)	-71.484*** (23.573)		58.749 (64.731)	32.218 (63.991)
R^2	0.293	0.167	0.377	0.275	0.033	0.285
Observations	61	61	61	23	23	23

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the usage level of internet per capita that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates over the sample period 1960-2002 for each country. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, ** indicate respectively statistical significance at the 1 and 5 percent level.

Table B.6: Volatility and Time Adoption Lags — Cell Phones per Capita

Independent Variable	Full Sample (68 countries, 1960-2002)		
	(1)	(2)	(3)
<i>Volatility</i>	31.623** (15.845)		40.749*** (13.664)
<i>Growth</i>		-143.486*** (30.864)	-152.402*** (20.906)
R^2	0.047	0.266	0.344
Observations	68	68	68

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the usage level of cell phones per capita that each country had in the benchmark year 2002. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates over the sample period 1960-2002 for each country. The OECD sample is not shown because it would include only four countries: time adoption lags for cell phones are censored because the U.S. never reaches the usage level that most OECD countries have in the benchmark year 2002. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. ***, ** indicate respectively statistical significance at the 1 and 5 percent level.

Table B.7: Volatility and Time Adoption Lags (Benchmark Year 1995)

Independent Variable	Computers (40 countries, 1973-1995)	Internet (19 countries, 1983-1995)	Cell Phones (47 countries, 1973-1995)
	(1)	(2)	(3)
<i>Volatility</i>	99.823*** (18.947)	30.688 (58.677)	60.307*** (15.845)
<i>Growth</i>	41.296 (29.880)	5.021 (37.541)	-31.489 (28.855)
R^2	0.334	0.049	0.283
Observations	40	19	47

Notes: The dependent variable *Time Adoption Lag* measures how many years ago the U.S. last had the technology usage level that each country had in the benchmark year 1995. *Growth* and *Volatility* are respectively the mean and standard deviation of per capita real GDP annual growth rates calculated over the sample period from the invention year of a specific technology to the benchmark year 1995. Heteroskedasticity robust standard errors in parentheses. Regressions include a constant. *** indicates statistical significance at the 1 percent level.

Appendix C

Appendix to Chapter 3

C.1 Firms' Behavior and the Free-Entry Equilibrium

To characterize the typical firm's behavior, consider the Current Value Hamiltonian (CVH, henceforth)

$$CVH_i = [P_i - C_X(W, P_M)Z_i^{-\theta}]X_i - W\phi - WL_{Z_i} + z_i\alpha KL_{Z_i},$$

where the costate variable, z_i , is the value of the marginal unit of knowledge. The firm's knowledge stock, Z_i , is the state variable; effort in cost reduction, L_{Z_i} , and the product's price, P_i , are the control variables. Firms take the public knowledge stock, K , as given. Since the Hamiltonian is linear, one has three cases: 1) $W > z_i\alpha K$ implies that the value of the marginal unit of knowledge is lower than its cost. The firm, then, does not invest; 2) $W < z_i\alpha K$ implies that the value of the marginal unit of knowledge is higher than its cost. Since the firm demands an infinite amount of labor to employ in cost reduction, this case violates the general equilibrium conditions and is ruled out; 3) the first order conditions for the interior solution are given by equality between marginal revenue and marginal cost of knowledge, $W = z_i\alpha K$, the

constraint on the state variable, (3.11), the terminal condition,

$$\lim_{s \rightarrow \infty} e^{-\int_t^s [r(v) + \delta] dv} z_i(s) Z_i(s) = 0,$$

and a differential equation in the costate variable,

$$r + \delta = \frac{\dot{z}_i}{z_i} + \theta C_X(W, P_M) Z_i^{-\theta-1} \frac{X_i}{z_i},$$

that defines the rate of return to cost reduction as the ratio between revenues from the knowledge stock and its shadow price plus (minus) the appreciation (depreciation) in the value of knowledge.

The revenue from the marginal unit of knowledge is given by the cost reduction it yields times the scale of production to which it applies. The price strategy is

$$P_i = C_X(W, P_M) Z_i^{-\theta} \frac{\epsilon}{\epsilon - 1}. \quad (\text{C.1})$$

Peretto (1998) (Proposition 1) shows that under the restriction $1 > \theta(\epsilon - 1)$ the firm is always at the interior solution, where $W = z_i \alpha K$ holds, and equilibrium is symmetric. The cost function (3.10) gives rise to the conditional factor demands:

$$L_{X_i} = \frac{\partial C_X(W, P_M)}{\partial W} Z_i^{-\theta} X_i + \phi;$$

$$M_i = \frac{\partial C_X(W, P_M)}{\partial P_M} Z_i^{-\theta} X_i.$$

Then, the price strategy (C.1), symmetry and aggregation across firms yields (3.13) and (3.14). Also, in symmetric equilibrium $K = Z = Z_i$ yields $\dot{K}/K = \alpha L_Z/N$, where L_Z is aggregate effort in cost reduction. Taking logs and time derivatives of $W = z_i \alpha K$ and using the demand curve (3.8), the cost-reduction technology (3.11) and the price strategy (C.1), one reduces the first-order conditions to (3.15).

Taking logs and time-derivatives of V_i yields

$$r = \frac{\Pi_i}{V_i} + \frac{\dot{V}_i}{V_i} - \delta.$$

The cost of entry is $\beta WY/N$. The corresponding demand for labor in entry is L_N . The case $V > \beta WY/N$ yields an unbounded demand for labor in entry, $L_N = +\infty$, and is ruled out since it violates the general equilibrium conditions. The case $V < \beta WY/N$ yields $L_N = -\infty$, which means that the non-negativity constraint on L_N binds and $L_N = 0$. A free-entry equilibrium requires $V = \beta WY/N$. Using the price strategy (C.1), the rate of return to entry becomes (3.16).

C.2 Proof of Proposition 12

Since the sectors producing the home consumption good and energy are competitive, we have $\Pi_H = \Pi_M = 0$. The consumption expenditure allocation rule (3.4) and the choice of numeraire yield

$$\dot{A} = rA + L + p\Omega - \frac{1}{\varphi}Y_H.$$

Rewriting the domestic commodity demand (3.19) as

$$pO = Y \cdot \xi(p), \quad \xi(p) \equiv \frac{\epsilon - 1}{\epsilon} S_X^M(p) S_M^O(p),$$

allows us to rewrite the balanced trade condition as

$$\frac{1}{\varphi}Y_H - p\Omega = Y(1 - \xi(p)).$$

Substituting the expressions for financial wealth, $A = \beta Y$, and the balanced trade condition in the household's budget constraint (3.3), and using the rate of return to

saving in (3.5), yields

$$\begin{aligned}\frac{\dot{Y}}{Y} &= \rho + \frac{\dot{Y}_H}{Y_H} + \frac{L + p\Omega - \frac{1}{\varphi}Y_H}{\beta Y} \\ &= \rho + \frac{\dot{Y}_H}{Y_H} + \frac{L - Y(1 - \xi(p))}{\beta Y}.\end{aligned}$$

Differentiating the balanced trade condition yields

$$\frac{1}{\varphi}\dot{Y}_H = \dot{Y}(1 - \xi(p)) \Rightarrow \frac{\dot{Y}_H}{Y_H} = \frac{\dot{Y}}{Y} \frac{Y}{Y_H} \varphi(1 - \xi(p)) = \frac{\dot{Y}}{Y} \frac{Y(1 - \xi(p))}{Y(1 - \xi(p)) + p\Omega}.$$

Substituting back in the budget constraint and rearranging terms yields

$$\frac{\dot{Y}}{Y} = \frac{Y(1 - \xi(p)) + p\Omega}{p\Omega} \left[\rho + \frac{L - Y(1 - \xi(p))}{\beta Y} \right].$$

This differential equation has a unique positive steady-state value of manufacturing production:

$$Y(p) = \frac{L}{1 - \xi(p) - \rho\beta}.$$

We ignore, for simplicity the issue of potential indeterminacy, assuming that Y jumps to this steady-state value. The associated expenditures on the home and foreign goods, respectively, are:

$$\begin{aligned}Y_H(p) &= \varphi \left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega \right]; \\ Y_F(p) &= (1 - \varphi) \left[\frac{L(1 - \xi(p))}{1 - \xi(p) - \rho\beta} + p\Omega \right].\end{aligned}$$

Since $Y_H(p)$ and $Y_F(p)$ are constant, the saving rule (3.5) yields that the interest rate is $r = \rho$ at all times.

C.3 Proof of Proposition 13

The return to entry (3.16) and the entry technology $\dot{N} = (N/\beta Y) \cdot L_N - \delta N$ yield

$$L_N = \frac{Y}{\epsilon x} \left[x - \left(\phi + \frac{L_Z}{N} \right) \right] - \rho \beta Y.$$

Taking into account the non-negativity constraint on L_Z , we solve (3.11) and (3.15) for

$$\frac{L_Z}{N} = \begin{cases} \theta(\epsilon - 1)x - (\rho + \delta)/\alpha & x > x_Z \equiv \frac{\rho + \delta}{\alpha\theta(\epsilon - 1)} \\ 0 & x \leq x_Z \end{cases}. \quad (\text{C.2})$$

Therefore,

$$L_N = \begin{cases} \frac{Y}{\epsilon} \left[1 - \theta(\epsilon - 1) - \frac{\phi - (\rho + \delta)/\alpha}{x} \right] - \rho \beta Y & x > x_Z \\ \frac{Y}{\epsilon} \left(1 - \frac{\phi}{x} \right) - \rho \beta Y & x \leq x_Z \end{cases}.$$

So we have

$$L_N > 0 \text{ for } \begin{cases} x > \frac{\phi - (\rho + \delta)/\alpha}{1 - \theta(\epsilon - 1) - \epsilon\rho\beta} & x > x_Z \\ x > \frac{\phi}{1 - \epsilon\rho\beta} & x \leq x_Z \end{cases}.$$

We look at the case

$$\frac{\phi}{1 - \epsilon\rho\beta} \equiv x_N < \frac{\rho + \delta}{\alpha\theta(\epsilon - 1)} \equiv x_Z,$$

which yields that the threshold for gross entry x_N is smaller than the threshold for in-house innovation x_Z .¹

To obtain the value of Y when $L_N = 0$, first note that

$$L_N = 0 \quad \text{for} \quad \frac{1}{\epsilon} \left(1 - \frac{\phi}{x} \right) \leq \rho\beta.$$

The household budget yields

$$0 = N \left(\frac{Y}{\epsilon N} - \phi \right) + L + \Omega p - \frac{1}{\varphi} Y_H.$$

¹ The global dynamics are well defined also when this condition fails and $x_N > x_Z$. We consider only the case $x_N < x_Z$ to streamline the presentation since the qualitative result and, most importantly, the insight about the role of the commodity price remain essentially the same.

Using the balanced trade condition and rearranging yields

$$Y = \frac{L - \phi N}{1 - \xi(p) - \frac{1}{\epsilon}}.$$

This equation holds for

$$x \leq x_N \equiv \frac{\phi}{1 - \epsilon \rho \beta} \Leftrightarrow N \geq N_N \equiv \frac{\phi}{1 - \epsilon \rho \beta} \frac{\epsilon}{Y}.$$

The interpretation is that with no effort in entry, there is net exit and thus saving of fixed costs. This shows up as aggregate efficiency gains as intermediate firms move down their average cost curves. Note that in this region,

$$Y(t) = \frac{L - \phi N_0 e^{-\delta t}}{1 - \xi(p) - \frac{1}{\epsilon}},$$

which shows that intermediate production grows in value as a result of net exit. The consolidation of the market results in growing profitability, that is,

$$\frac{\dot{x}}{x} = \frac{\delta L}{L - \phi N_0 e^{-\delta t}} \Rightarrow \dot{x} = \frac{\delta L / \epsilon N_0}{1 - \xi(p) - \frac{1}{\epsilon}}.$$

This says that with the exit shock, the economy must enter the region where entry is positive because the very definition of steady state requires replacing firms that leave the market. Therefore, the only condition that we need to ensure convergence to the steady state with positive cost reduction is $x^* > x_Z$.

C.4 Proof of proposition 14

Taking logs of (3.26) yields

$$\log T(t) = \theta \log Z_0 + \theta \int_0^t \hat{Z}(s) ds + \chi \log N_0 + \chi \log \left(\frac{N(t)}{N_0} \right).$$

Using the expression for g in (3.27), and adding and subtracting \hat{Z}^* from $\hat{Z}(t)$, we obtain

$$\log T(t) = \log(Z_0^\theta N_0^\chi) + gt + \theta \int_0^t [\hat{Z}(s) - \hat{Z}^*] ds + \chi \log\left(\frac{N(t)}{N_0}\right).$$

Using (C.2) and (3.28) we rewrite the third term as

$$\begin{aligned} \theta \int_0^t (\hat{Z}(s) - \hat{Z}^*) ds &= \alpha \theta^2 (\epsilon - 1) \int_0^t (x(s) - x^*) ds \\ &= \gamma \left(\frac{x_0}{x^*} - 1 \right) \int_0^t e^{-\nu s} ds \\ &= \frac{\gamma}{\nu} \left(\frac{x_0}{x^*} - 1 \right) (1 - e^{-\nu t}), \end{aligned}$$

where

$$\gamma \equiv \alpha \theta^2 (\epsilon - 1) x^*.$$

Observing that $N(t) = Y(p)/\epsilon x(t)$ yields $\dot{N}/N = -\dot{x}/x$, we use (3.28) to obtain

$$\frac{N(t)}{N_0} = \frac{1 + \left(\frac{N^*}{N_0} - 1\right)}{1 + \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t}}.$$

We then rewrite the last term as

$$\begin{aligned} \chi \log\left(\frac{N(t)}{N_0}\right) &= \chi \log \frac{1 + \left(\frac{N^*}{N_0} - 1\right)}{1 + \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t}} \\ &= \chi \log\left(1 + \left(\frac{N^*}{N_0} - 1\right)\right) - \chi \log\left(1 + \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t}\right). \end{aligned}$$

Approximating the log terms, we can write

$$\begin{aligned} \chi \log\left(\frac{N(t)}{N_0}\right) &= \chi \left(\frac{N^*}{N_0} - 1\right) - \chi \left(\frac{N^*}{N_0} - 1\right) e^{-\nu t} \\ &= \chi \left(\frac{N^*}{N_0} - 1\right) (1 - e^{-\nu t}). \end{aligned}$$

Observing that

$$\frac{N^*}{N_0} - 1 = \frac{x_0}{x^*} - 1,$$

these results yield (3.29).

Now consider

$$\begin{aligned} \log u &= \varphi \log \left(\frac{Y_H}{P_H L} \right) + (1 - \varphi) \log \left(\frac{Y_F}{P_F L} \right) \\ &= \varphi \log \left(\frac{Y_H}{P_H L} \right) + (1 - \varphi) \log \left(\frac{\frac{1-\varphi}{\varphi} Y_H}{P_F L} \right) \\ &= \log \left(\frac{Y_H}{L} \right) - \varphi \log P_H + (1 - \varphi) \log \left(\frac{1 - \varphi}{\varphi P_F} \right) \\ &= \log \left(\frac{Y_H}{L} \right) - \varphi \log c(p) + \varphi \log T - \varphi \log \left(\frac{\epsilon}{\epsilon - 1} \right) + (1 - \varphi) \log \left(\frac{1 - \varphi}{\varphi P_F} \right). \end{aligned}$$

To simplify the notation, and without loss of generality, we set

$$(1 - \varphi) \log \left(\frac{1 - \varphi}{\varphi P_F} \right) + \varphi \log (N_0^\chi Z_0^\theta) - \varphi \log \left(\frac{\epsilon}{\epsilon - 1} \right) \equiv 0.$$

This is just a normalization that does not affect the results. We then substitute the expression derived above into (3.1) and write

$$\begin{aligned} U(p) &= \int_0^\infty e^{-\rho t} \left[\log \varphi \left(\frac{1 - \xi(p)}{1 - \xi(p) - \rho\beta} + \frac{p\Omega}{L} \right) - \varphi \log (c(p)) + \varphi g t \right] dt \\ &\quad + \varphi \left(\frac{\gamma}{\nu} + \chi \right) \Delta \int_0^\infty e^{-\rho t} (1 - e^{-\nu t}) dt. \end{aligned}$$

Integrating, we obtain (3.31).

C.5 Proof of Lemma 15

Observe that

$$\epsilon_X^M \equiv -\frac{\partial \log M}{\partial \log P_M} = 1 - \frac{\partial \log S_X^M}{\partial \log P_M} = 1 - \frac{\partial S_X^M}{\partial P_M} \frac{P_M}{S_X^M}$$

so that $\epsilon_X^M \leq 1$ if

$$\frac{\partial S_X^M}{\partial P_M} = \frac{\partial}{\partial P_M} \left(\frac{P_M M}{P_M M + L_X} \right) \geq 0.$$

This in turn is true if

$$(1 - S_X^M) \frac{\partial (P_M M)}{\partial P_M} - S_X^M \frac{\partial L_X}{\partial P_M} \geq 0.$$

Recall now that total cost is increasing in P_M so that

$$\frac{\partial (P_M M)}{\partial P_M} + \frac{\partial L_X}{\partial P_M} > 0 \Rightarrow \frac{\partial (P_M M)}{\partial P_M} > -\frac{\partial L_X}{\partial P_M}.$$

It follows that

$$\frac{\partial L_X}{\partial P_M} \leq 0$$

is a sufficient condition for $\epsilon_X^M \leq 1$ since it implies that both terms in the inequality above are positive. The proof for $\epsilon_M^O \leq 1$ is analogous.

C.6 Proof of Proposition 17

(3.19) and (3.20) yield

$$\Omega \geq O \Leftrightarrow \frac{\Omega}{L} \geq \frac{1}{1 - \xi(p) - \beta\rho} \frac{\xi(p)}{p}.$$

Differentiating (3.20) yields

$$\frac{d \log Y(p)}{dp} = -\frac{d \log (1 - \xi(p) - \beta\rho)}{dp} = \frac{\xi'(p)}{1 - \xi(p) - \beta\rho}.$$

It is useful to write

$$\xi'(p) = \frac{\xi(p)}{p} \left[(1 - \epsilon_X^M(p)) S_M^O(p) + 1 - \epsilon_M^O(p) \right],$$

which shows that the sign of $\xi'(p)$ depends on the upstream and downstream price elasticities of demand and on the overall contribution of the commodity to manufacturing cost. Assume for example that $1 - \epsilon_X^M(p) < 0$ and $1 - \epsilon_M^O(p) > 0$ because the upstream, materials technology exhibits labor-commodity complementarity and the downstream, manufacturing technology exhibits labor-materials substitution. Then there exists a price p^v such that

$$\xi'(p) = \frac{\xi(p)}{p} \left[(1 - \epsilon_X^M(p)) S_M^O(p) + 1 - \epsilon_M^O(p) \right] = 0.$$

That is,

$$(\epsilon_X^M(p) - 1) S_M^O(p) = 1 - \epsilon_M^O(p).$$

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Biography

My name is Domenico Ferraro. I was born on June 5 in Napoli, Italy. I have earned a Bachelor of Arts degree in Economics and a Master of Arts in Economics and Finance from Federico II University in Napoli, Italy. I also hold a Master of Science in Economics from Universitat Pompeu Fabra in Barcelona (Spain) and a Ph.D. in Economics from Duke University. I have recently accepted an offer as Assistant Professor of Economics at Arizona State University (ASU) starting in August 2014.